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Rotary Wing Aeroelasticity—A Historical Perspective

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This paper provides a historical perspective of the fundamental developments that have played a central role in rotary-wing dynamics and aeroelasticity and have had a major impact on the design of rotary-wing aircraft. The paper describes a historical progression starting with the classical flap-pitch problem that emulated fixed-wing behavior and describes the evolution of the dynamic and aeroelastic problems into those that are unique to rotorcraft, such as the flap-lag problem, the lag-pitch problem, and the coupled flap-lag-torsional problem. Subsequently, the coupled rotor/fuselage aeromechanical problems such as ground and air resonance are considered. A description of the evolution of the methodology used in the formulation and solution of these types of problems is also provided, emphasizing the structural and aerodynamic models required for their effective formulation and solution. The mathematical techniques used for solving the rotary-wing aeroelastic problems in hover and forward flight are also described. The primary emphasis of the paper is on aeroelastic stability, and aeroelastic response is only treated briefly. The paper focuses on contributions that have historical value because they represent landmark treatments. Because of the large amount of material available, an all-inclusive treatment of the research done in this field is impractical, and the paper has unavoidable omissions.

Nomenclature

$\boldsymbol{a}$ = acceleration vector
$[B(\psi)]$ = tranformation matrix for multiblade coordinates
b = semichord
C_{d0} = profile drag coefficient
$C(k)$ = Theodorsen's lift deficiency function
$C'(k, m, \bar{h}_w)$ = Loewy's lift deficiency function
C_W = weight coefficient
$[C(\psi)]$ = symbolic matrix, representing linear damping effects
$\hat{\boldsymbol{e}}_x, \hat{\boldsymbol{e}}_y, \hat{\boldsymbol{e}}_z$ = unit vectors in the directions of the coordinates, x_0, y_0, z_0, respectively before deformation
$\hat{\boldsymbol{e}}'_x, \hat{\boldsymbol{e}}'_y, \hat{\boldsymbol{e}}'_z$ = triad $\hat{\boldsymbol{e}}_x, \hat{\boldsymbol{e}}_y, \hat{\boldsymbol{e}}_z$ after deformation
e_1 = offset of blade root from axis of rotation

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$\{F_{NL}(\psi, \boldsymbol{q}, \dot{\boldsymbol{q}}\}$	=	complete nonlinear state vector loading
h	=	plunging motion, used in unsteady aerodynamics
$\bar{h}_w$	=	(h_w/b) nondimensional wake spacing
I_ζ	=	blade inertia about lag hinge
k; $(\omega b/U)$	=	reduced frequency
L	=	unsteady lift, per unit length based on Greenberg's theory
$[L(\psi)]$	=	linear coefficient matrix
l	=	length of elastic part of the blade
m	=	(ω/Ω) frequency ratio
N or N_b	=	number of blades
$\{N(\boldsymbol{q}, \psi)\}$	=	nonlinear vector
$\boldsymbol{q}$	=	unknown state vector
R	=	blade radius
$\boldsymbol{R}$	=	position vector of a mass point of blade cross section, in blade-fixed, rotating reference frame
$[S]$	=	transformation matrix between triads $(\hat{\boldsymbol{e}}_x, \hat{\boldsymbol{e}}_y, \hat{\boldsymbol{e}}_z)$ and $(\hat{\boldsymbol{e}}'_x, \hat{\boldsymbol{e}}'_y, \hat{\boldsymbol{e}}'_z)$
u, v, w	=	components of the displacement of a point on the elastic axis of the blade in directions, $\hat{\boldsymbol{e}}_x$, $\hat{\boldsymbol{e}}_y$, and $\hat{\boldsymbol{e}}_z$, respectively, subscript k implies kth blade
V	=	pulsating flow velocity in Greenberg's theory
ΔV	=	varying part of V
V_0	=	constant part of V
v_i	=	mean induced velocity at the rotor disc
$\{\boldsymbol{X}\}$	=	generalized coordinate vector
x_A	=	blade cross-sectional aerodynamic center (A.C.) offset from elastic axis (E.A.), positive for A.C. before E.A.
$\{z(\psi)\}$	=	known periodic forcing
α_0	=	constant part of pitch, or angle of attack
β	=	flap angle
β_p	=	preconing, inclination of the feathering axis with respect to the hub plane measured in a vertical plane
β_0	=	steady flap angle
β_1, β_2	=	rigid-body flapping angle for teetering rotor
γ	=	Lock number
$\Delta\alpha$	=	change in angle of attack for dynamic stall
$\delta\lambda$	=	perturbation in steady inflow ratio
ϵ	=	basis for order of magnitude, associated with typical elastic blade slopes
ζ	=	lag angle
η_{SLi}	=	viscous structural damping coefficients in percent of critical damping, for the lag modes
θ	=	total pitch angle
θ_0	=	steady pitch angle
θ_{1s}, θ_{1c}	=	cyclic pitch components
$\bar{\lambda}$	=	constant part of the inflow ratio
λ_{1s}, λ_{1c}	=	cyclic components of inflow ratio
μ	=	$V\cos\alpha_R/\Omega R$ advance ratio
ρ_A	=	density of air
σ	=	blade solidity ratio: blade area/disk area
ϕ	=	rotation of a cross section of the blade around the elastic axis
ψ	=	azimuth angle of blade ($\psi = \Omega t$) measured from straight aft position
$\boldsymbol{\Omega}$	=	angular velocity vector
$\bar{\omega}_{F1}$, $\bar{\omega}_{L1}$, $\bar{\omega}_{T1}$	=	First rotating natural frequencies in flap, lag, and torsion, respectively, nondimensionalized with respect to Ω
ω_θ	=	torsional frequency

I. Introduction and Background

THE 100th anniversary of the Wright brothers' historic flight is being celebrated by a variety of events, and several survey papers dealing with various aspects of aeroelasticity are also being written for this occasion. The present paper focuses on rotary-wing aeroelasticity. Its objective is to provide a historical perspective on this fascinating field.

When reviewing research in rotary-wing aeroelasticity (RWA), it is important to note a few historical facts. The Wright brothers flew in 1903, and Sikorsky built and started flying the first operational helicopter, the R-4 or (VS-316), in 1942. The R-4 was a three-bladed helicopter with a rotor diameter of 11.6 m and was powered by a 185-hp engine. Thus, there is an initial gap of 39 years between fixed-wing and rotary-wing technologies. Therefore, it is not surprising that certain rotary-wing problems, particularly those pertaining to unsteady aerodynamics, are still not well understood. The situation is further compounded by the complexity of the vehicle when compared to fixed-wing aircraft.

The field of rotary-wing aeroelasticity has been a very active area of research during the last 40 years. This research activity has generated a large number of papers, which combined with the papers in this area published between 1945–1963, constitutes a large body of literature that is impossible to review in a single survey paper. Fortunately, a considerable number of review papers and books have also been published.

These review papers, when considered in chronological order, provide a historical perspective on the evolution of the field.[1–14] One of the first significant reviews of rotary-wing dynamic and aeroelastic problems was provided by Loewy,[12] where a wide range of dynamic problems was reviewed in considerable detail. A more limited survey emphasizing the role of unsteady aerodynamics and vibration problems in forward flight was presented by Dat.[2] Two comprehensive reviews of rotary-wing aeroelasticity were presented by Friedmann.[3,4] In Ref. 3 a detailed chronological discussion of the flap-lag and coupled flap-lag-torsion problems in hover and forward flight was presented, emphasizing the inherently nonlinear nature of the hingeless blade aeroelastic stability problem. The nonlinearities considered were geometrical nonlinearities caused by moderate blade deflections. In Ref. 4, the role of unsteady aerodynamics, including dynamic stall, was examined, together with the treatment of nonlinear aeroelastic problems in forward flight. Finite element solutions to RWA problems were also considered, together with the treatment of coupled rotor-fuselage problems. Another detailed survey by Ormiston[13] discussed the aeroelasticity of hingeless and bearingless rotors, in hover, from an experimental and theoretical point of view.

Although aeroelastic stability plays an important role in the design of rotor systems, the aeroelastic response problem as represented by the rotorcraft vibration and dynamic loads prediction plays an even more critical role. Thus, two other surveys have dealt exclusively with vibration and its control in rotorcraft.[15,16] These papers focus on the vibrations caused by the aeroelastic response of the rotor, and the study of various passive, semiactive, and active devices for controlling such vibrations.

Johnson[10,11] has published a comprehensive review paper addressing both aeroelastic stability and vibration problems for advanced rotor systems. In a sequel[5] to his previous review papers, Friedmann discussed the principal developments that have taken place between 1983–1987, emphasizing new methods for formulating aeroelastic problems, advances in treatment of the aeroelastic problem in forward flight, coupled rotor-fuselage analyses, structural blade modeling, structural optimization, and the use of active control for vibration reduction and stability augmentation.

A comprehensive report,[14] which contains a detailed review of the theoretical and experimental development in the aeroelastic and aeromechanical stability of helicopters and tilt-rotor aircraft, carried out under U.S. Army/NASA sponsorship during the period 1967–1987 was prepared by Ormiston et al. Somewhat later, key ideas and developments in four specific areas— 1) role of geometric nonlinearities in RWA, 2) structural modeling of composite blades, 3) coupled rotor-fuselage aeromechanical problems and their active control, and 4) higher harmonic control for vibration reduction in rotorcraft—were considered by Friedmann.[6] At the same time

Chopra[1] surveyed the state of the art in aeromechanical stability of helicopters, including pitch flap, flap lag, coupled flap lag torsion, air and ground resonance. Advances in aeromechanical analysis of bearingless, circulation-controlled, and composite rotors were also treated in this detailed paper. Perhaps the most comprehensive paper on RWA was written by Friedmann and Hodges.[9] This paper contains close to 350 references and dwells on all of the important aspects of rotary-wing aeroelastic stability and response problems. The treatment is broad and comprehensive and is current up to 1991. A partial review of some recent developments can also be found in Ref. 7.

In addition to the numerous papers dealing with the subject of this review, this topic is also described in a number of books. Among these, the most notable one is Johnson's[17] monumental treatise on helicopter theory, which contains extensive, detailed, and useful material on aerodynamic, dynamic, and mathematical aspects of rotary-wing aerodynamics, dynamics, and aeroelasticity. A more recent book[18] treats several aeroelastic and structural dynamic problems in rotorcraft. Quite recently, Leishman[19] has written an excellent book on helicopter aerodynamics, which contains good treatments of unsteady aerodynamics, rotor wake models, and dynamic stall.

The principal objectives of this paper are as follows:

1) Present the historical evolution of modern rotary-wing aeroelasticity, starting with the isolated blade aeroelastic problem and progressing to the coupled rotor fuselage aeromechanical problem.

2) Present the evolution of the methodology for formulation and solution of rotary-wing aeroelastic problems. The principal focus will be on aeroelastic stability; therefore, the aeroelastic response problem will be mentioned only briefly.

3) Describe some current trends so as to illustrate considerable differences between current and past endeavors.

The paper will not attempt to provide a comprehensive literature review of all of the papers published in the field. Instead, it will focus on particular studies that have a historical value because they represent an important contribution to the field of RWA.

To understand the historical development of RWA, it is important to recognize that the mathematical models capable of simulating rotary-wing aeroelastic behavior were intimately linked to the types of helicopter rotors used. The evolution of the various types of main rotor systems was the principal driver that provided the impetus for the development of the mathematical modeling tools. The first generation of helicopters used articulated blades. A typical articulated rotor hub together with an idealized representation for mathematical modeling are shown in Fig. 1. For this class of rotors, the dynamics of the blade are characterized by the flap β, lag ζ, and pitch θ angles, which allow the blade to move as a rigid body. Flexible bending and torsional displacement can be added to the displacements as a result of the rigid-body motion.

A few years later teetering rotors, shown in Fig. 2, were developed and used extensively on helicopters manufactured by Bell as well

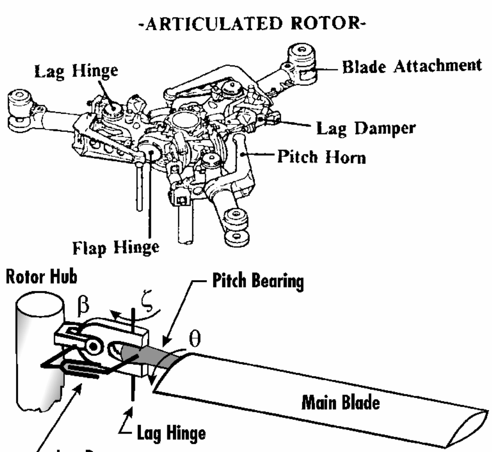

Fig. 1 Typical articulated hub (top) and typical articulated blade model (bottom).

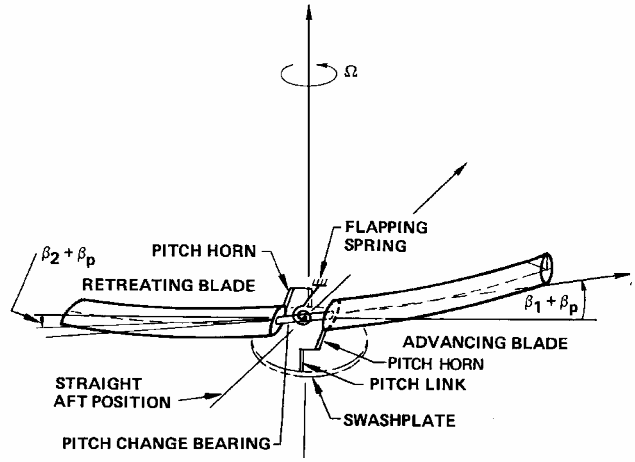

Fig. 2 Typical teetering blade model.

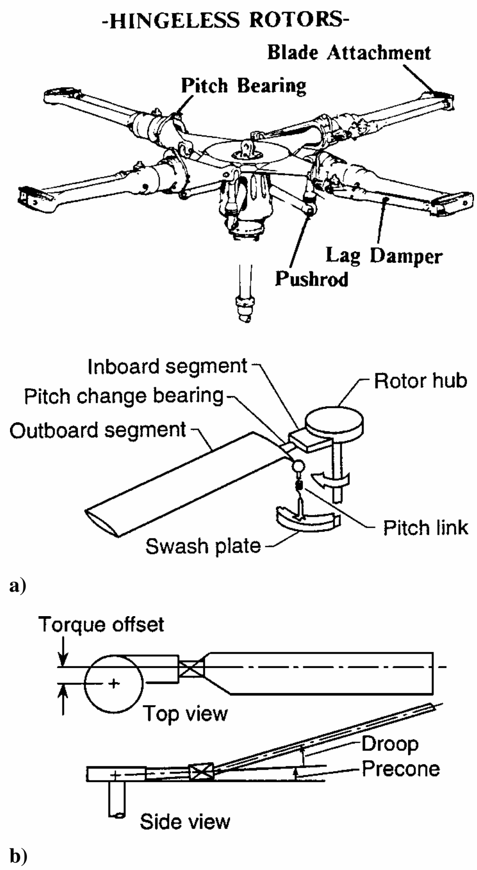

Fig. 3 Typical hingeless rotor hub (top) and two views of a typical hingeless blade used in mathematical modeling (bottom).

as other companies. These blades also have a flapping hinge, except that now the rigid-body flap angle on the first blade is equal and opposite to that on the second blade, that is, $\beta_1 = -\beta_2$; elastic flap, lag, and torsional deformation can be superimposed on the rigid-body flapping motion. Teetering rotors were suitable primarily for lighter helicopters because the size of the blades for heavy helicopters creates almost insurmountable dynamic problems.

The next step in the evolution of rotor systems was the development of the hingeless rotors shown in Fig. 3. Hingeless rotor configurations started appearing in the early 1960s and became operational in the late 1960s and early 1970s. Figure 3 depicts a typical example of a hingeless hub together with a typical model for a hingeless blade. These blades have no flap or lag hinges. The pitch bearing is still needed to introduce the collective and cyclic components of pitch.

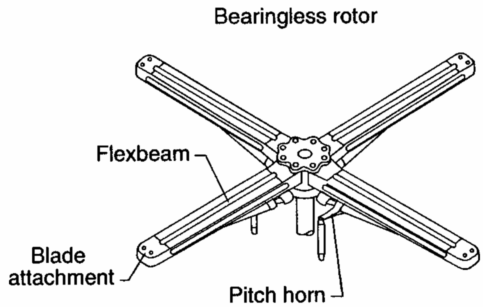

Fig. 4 Typical bearingless rotor hub.

The final step in the evolution of main rotor systems is the bearingless rotor depicted in Fig. 4. Bearingless rotor configurations started appearing in the late 1960s and early 1970s. However, they were incorporated in helicopters that went into production only in the late 1990s. This rotor has no hinges; both the flap and lag degrees of freedom are cantilevered. The pitch bearing is replaced by a flexbeam, and the pitch inputs to the blade are provided by elastically twisting the blade using the pitch horn.

With this background it is now possible to review some of the most important developments in RWA. For convenience, the time period from the mid 1940s to the present is divided into three principal periods: 1) the early years, 1945–1970, when engineers and researchers were struggling to accommodate new developments in rotor hardware; 2) the golden age, 1970–2000, when many important contributions were made leading to a much better understanding of the methods for formulating and solving the RWA problem; and 3) the 21st century or period of refinement, 2000–present, when the large computing power currently available is utilized to refine the accuracy and reliability of the methods for formulating and solving aeroelastic problems, by introducing computational aeroelasticity and combining it with control, acoustics, and optimization in a more general aeromechanical framework.

Each of these periods is considered in detail in the following sections.

II. Early Years (1945–1970)

A. Isolated Blade Stability

The state of the art emerges when reading all of the papers published during this time period. However, an excellent description of this period can be found in Loewy's outstanding survey paper.[12] The insight provided by Loewy is augmented by several other surveys that partially cover this time period.[2,3,20] This was an interesting period characterized by rapid hardware developments combined with a lack of sophisticated models capable of replicating the aeroelastic behavior. The appropriate methodology for formulating and solving the rotary-wing behavior was not well understood, and the field was strongly influenced by the desire to adapt the most successful tools that have proven themselves for the fixed-wing static and dynamic aeroelastic problems to the rotary-wing case. Since the majority of the rotor systems were articulated, the analyses developed were aimed at modeling the blade configuration shown in Fig. 1.

A landmark contribution in this area was a paper by Miller and Ellis.[21] The formulation of the problem was carried out by using the direct Newtonian approach and writing the equations of moment equilibrium about the hinge. An important facet of this paper, which was somewhat typical also of other papers generated in this period, was the fact that the individuals associated with the work had industrial experience and outstanding intuitive understanding of the physics of the problem. Thus, even without achieving a completely accurate formulation (i.e., some terms in the equations could be missing, but they were usually quite small) the conclusions and the insight provided were usually quite accurate.

The basic problem treated was the coupled flap-pitch problem, with β and θ degrees of freedom shown in Fig. 1, augmented by blade elastic bending. For aeroelastic stability the emphasis was on hover, using unsteady aerodynamics that represented essentially a quasi-steady version of Theodorsen theory.[22] This resembled the classical bending-torsion flutter analysis of a fixed wing, augmented by the aerodynamic and inertia terms caused by rotation. Blade stability was determined from linear constant coefficient equations, which resembled the small perturbation equations commonly used in fixed-wing aeroelasticity.

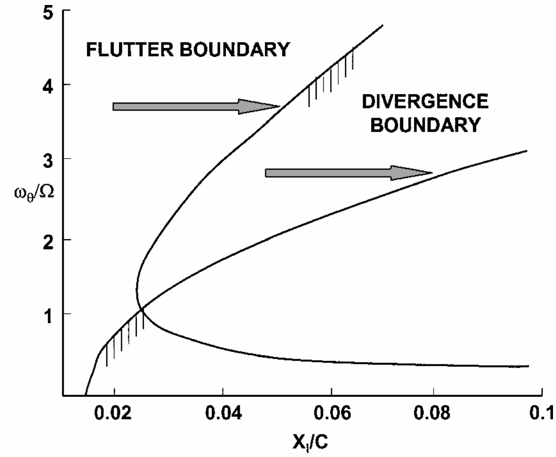

Fig. 5 Typical flap-pitch stability boundaries, showing divergence and flutter as a function of blade c.g. offset from feathering axis x_I/c, $\gamma = 12$, $\omega_\beta = 1$, and $c/R = 0.05$.

A typical stability boundary associated with this type of analysis is shown in Fig. 5. The stability boundary is plotted by providing the torsional stiffness (ω_θ/Ω) in per rev, plotted against the offset of the cross-sectional center of gravity behind the feathering axis. Several interesting aspects are noteworthy. Both divergence and flutter boundaries are evident. Divergence depends on the offset between feathering axis and cross-sectional c.g. offset. This differs from fixed-wing divergence, which depends strictly on offset between elastic axis and aerodynamic center. It can be shown that Theodorsen-type unsteady aerodynamics has only a minor effect on the flutter boundary because the reduced frequency k_e is low.[21] Two other important effects identified in Ref. 21 were the effect of steady coning and steady in-plane bending. It was noted[21] that steady elastic flapping deflections have a minor effect on blade stability for an articulated rotor. On the other hand, it was emphasized that steady elastic in-plane deflection can have a major effect on blade stability, particularly for nonuniform spanwise mass distribution.[21] Therefore, this was one of the first studies to pinpoint the significance of the lag degree of freedom in rotary-wing aeroelastic stability.

The type of stability boundary shown in Fig. 5 can be modified significantly by kinematic coupling K_p between the flap and feathering degrees of freedom as shown in Refs. 23 and 24. Pitch-flap coupling can be introduced by a skewed flap-hinge geometry relative to the radial axis of the blade as shown in Fig. 6a, or by an appropriate positioning of the pitch link relative to the flap hinge as shown in Fig. 6b. The pitch-flap coupling is represented by

$$\Delta\theta = -K_p \Delta\beta \tag{1}$$

for the geometry shown in Fig. 6a, $K_p = \tan\delta_3$, $K_p > 0$, flap up decreases the blade pitch. Positive pitch-flap coupling acts as an aerodynamic spring on the flap motion and has a significant influence on flap-pitch stability.

As mentioned, the importance of large steady in-plane deflection on flap-pitch instabilities was identified in Ref. 21. Thus, it was only natural that the next type of instability to receive attention was the pitch-lag instability.

The first comprehensive study of the pitch-lag instability was carried out by P. C. Chou.[25,26] This instability was encountered during the whirl tower testing of a very light rotor blade designed by the Prewitt Aircraft Company for the Vertol H-21 helicopter. High-amplitude oscillation occurred at low Ω, high collective and maximum power, primarily in lead lag, at a frequency of 0.318/rev (close to lag frequency) and lag amplitude of 30 deg. No coupling

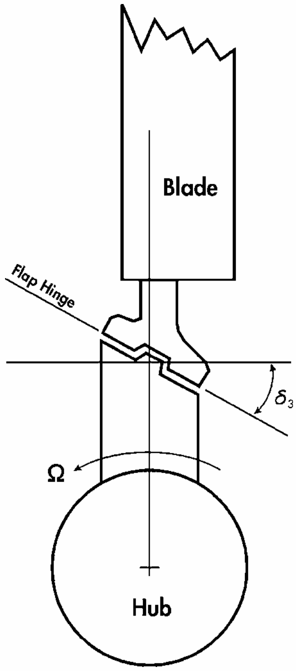

Fig. 6a Pitch-flap coupling caused by skewed flap hinge.

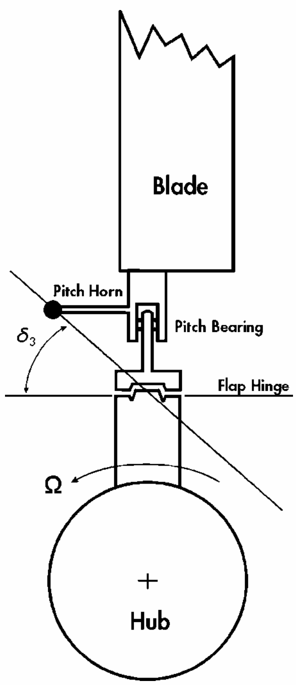

Fig. 6b Pitch-flap coupling.

between rotor and tower dynamics was found, and despite the large oscillations the blades sustained no damage.

A comprehensive analysis of this instability for hover was developed by Chou.[25,26] The analysis was linear and restricted to fully articulated rotors with inelastic blades. A lag damper assumed to have constant viscous damping C_ζ was included in the analysis. It was found that the instability was caused by pitch-lag coupling

$$\Delta\theta = -K_L \Delta\zeta \tag{2}$$

introduced by skewed lag hinges located outboard of the flapping hinge. An elegant approximate stability criterion was obtained from the analytical model

$$C_\zeta + \frac{2K_L\beta_0^2\Omega I_\zeta}{[1-(\beta_0/\theta_0)K_p]\theta_0} > 0 \tag{3}$$

which facilitated the design of stable blades.

During the mid-1960s, two new types of rotor systems, tilt rotors and hingeless rotors, emerged. The modeling of this class of rotor systems started a lengthy preoccupation with one of the most interesting and vexing dynamic problems, the coupled flap-lag aeroelastic problem. The first paper attempting to develop a model for the flap-lag instability for hingeless and teetering rotors was presented by Young.[27] The equations of motion for hover and forward flight were derived in an ad hoc manner. The author recognized that to capture the mechanism of instability the coupling between the flap and lag degrees of freedom, caused by aerodynamic and coriolis effects, is required. Because these two types of terms are nonlinear, they were included, but not in a consistent manner, that is, whereas terms having a certain order of magnitude were included, others having a similar order of magnitude were missing in the equations of motion. The effects of elastic modes and advance ratio were also incorporated in an approximate manner. Inspired by the stability criterion shown in Eq. (3), the author derived a fairly complicated stability criterion for the flap-lag case for both hover and forward flight. Using this stability criterion, a number of sweeping conclusions were reached, some of these were incorrect, some partially correct, and a few were correct. The paper correctly identified the lag degree of freedom as the trigger for the flap-lag instability, and it also identified the aerodynamic and inertial coupling terms as important. However, the stability criterion was false; and, therefore, the conclusion that "...all current rotor types are susceptible [to instability] in the speed range of 125–150 knots, or at lower speeds at high altitude..." was also incorrect.[27] Subsequently, Hohenemser and Heaton[28] treated the same problem using a different formulation, which suffered from inaccuracies similar to Young's caused by a variety of approximations. Instead of a stability criterion, they tried to determine blade stability by using a somewhat unconventional numerical integration scheme. The results presented in the paper were mainly of a qualitative nature.

Both studies failed to clearly identify the nature of the flap-lag instability problem because they did not account for the critical role of the elastic or structural coupling between the flap and lag degrees of freedom. In retrospect, this is somewhat surprising because a monumental NASA technical report written by Houbolt and Brooks[29] was available at that time and it contained the correct structural coupling terms which were required for the proper treatment of this problem. It was important to mention that Ref. 29 was overlooked by many studies on RWA conducted during this time period, and its value was only recognized belatedly, in the late 1960s and early 1970s.

It is remarkable that while treatments of flap pitch, pitch lag, and even flap lag were presented for the case of hover a comprehensive analysis of coupled flap-lag-torsional blade stability in hover failed to materialize. Although a set of suitable equations were derived in Ref. 30, numerical results illustrating blade aeroelastic behavior in hover were not computed.

Up to this point, the aeroelastic problems discussed were mainly those associated with the hovering flight condition, which is governed by differential equations with constant coefficients. One of the earliest papers to recognize the effect of periodic coefficients caused by forward flight on flapping motion was Horvay.[31] The periodic equations were solved using Hill's method of infinite determinants. Clearly, because only the flapping degree of freedom was considered the level of parametric excitation that is necessary to cause an instability had to be quite large. This in turn leads to very high advance ratios that do not occur during normal operating conditions of rotors in forward flight unless one slows the rotor down. This approach to dealing with the effect of periodic coefficients was used in the coupled flap-lag-torsional analysis of rotor blades presented in Bielawa's dissertation.[30] However, the numerical results obtained were inconclusive.

The studies considered up to now were based on quasi-steady or unsteady aerodynamic models that were developed essentially for the fixed-wing aeroelastic problem. However, there was a growing awareness that RWA requires unsteady aerodynamic models capable of representing the complicated aerodynamic environment present on a helicopter. The first important rotary-wing unsteady aerodynamic theory developed for hover is the work of Loewy.[32] This

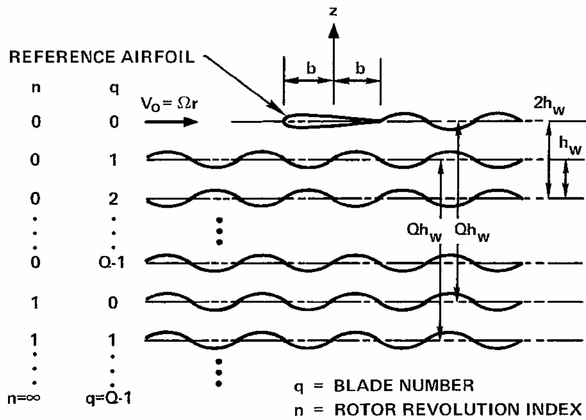

Fig. 7 Idealized wake geometry for Loewy's incompressible unsteady aerodynamic model.

theory is a generalization of Theodorsen's theory, and it provides a useful approximation to the unsteady wake beneath the hovering rotor. The geometry for Loewy's model is illustrated by Fig. 7. In this theory the effect of the spiral returning wake beneath the rotor is taken into account approximately. The wakes, infinite in number, lie in planes parallel to the disc of the rotor and are associated with both previous blades (for an N-bladed rotor) and previous revolutions. The nondimensional wake spacing $\bar{h}_w = (2\pi v_i/\Omega Nb) = 4\bar{\lambda}/\sigma$.

The airfoil dynamics in this theory are identical to the simple harmonic pitch-and-plunge motion postulated in Theodorsen's theory. Loewy has shown that for this case the unsteady aerodynamic lift and moment can be written in a form identical to Theodorsen's theory, except that Theodorsen's lift deficiency function $C(k)$ is replaced by a more complicated lift deficiency function given by $C'(k, m, \bar{h}_w)$.

Loewy's theory is restricted to low inflow ratios, which implies a lightly loaded disc. This theory was used for the first time to study "wake flutter" in Refs. 23 and 24, and the classical flap-pitch stability boundary shown in Fig. 5 is modified by several narrow instability regions present above the flutter boundary shown in Fig. 5.

Another useful aerodynamic theory developed in this time period was Greenberg's theory.[33] The theory recognizes that in addition to constant velocity of oncoming flow the blade can also experience a time-dependent, pulsating velocity variation caused by in-plane motion (lead lag). Furthermore, in addition to harmonic variation in angle of pitch a constant pitch angle is also imposed on the airfoil. Greenberg's theory is a modification to Theodorsen's theory to account for these effects. Thus, the unsteady lift on the blade cross section is given by

$$
\begin{aligned}
L = {} & \frac{1}{2}\rho_A a b^2 \left[\frac{\mathrm{d}^2 h}{\mathrm{d}t^2} + (V_0 + \Delta V)\frac{\mathrm{d}\Delta\alpha}{\mathrm{d}t} \right. \\
& \left. + (\alpha_0 + \Delta\alpha)\frac{\mathrm{d}(\Delta V)}{\mathrm{d}t} - \left(x_A - \frac{b}{2}\right)\frac{\mathrm{d}^2\Delta\alpha}{\mathrm{d}t^2} \right] \\
& + \rho_A a V b C(k) \left[\frac{\mathrm{d}h}{\mathrm{d}t} + \alpha_0 \Delta V + V_0 \Delta\alpha + (b - x_A)\frac{\mathrm{d}\Delta\alpha}{\mathrm{d}t} \right] \\
& + \rho_A a V b [\underline{V_0 \alpha_0} + \underbrace{\Delta\sigma V C(2k)}] \qquad (4)
\end{aligned}
$$

where $V = V_0 + \Delta V$; $\Delta V = \sigma_V V_0 e^{i\omega t}$; and α_0 = constant pitch setting.

The last two terms in this theory represent, respectively, the static lift (underlined) and a nonlinear term in the perturbation quantities (underbraced), which is usually neglected in rotary-wing applications of this theory. Greenberg's theory is approximate because it neglects the effect of fore and aft excursions of the blade or the effect of the pulsating flow velocity relative to the mean velocity on the wake. Reference 33 also provides an appropriate expression for the moment. Although Loewy's theory was applied to RWA aeroelastic problems shortly after its initial development, Greenberg's theory was not used until the mid-1970s, when its value was finally recognized.

Another concern associated with aerodynamic loading that materialized during this period was stall flutter. An important early investigation of stall flutter was conducted by Ham.[34] Retreating blade stall on a model rotor in forward flight was considered, and large torsional motion with a frequency close to the blade torsional natural frequency was found after the blade entered the stall region. The sensitivity of the blade torsional amplitude to several parameters was studied. Increases in speed and rearward shift of the blade cross-sectional center of gravity caused increases in the amplitude of torsional oscillation. However, increases in torsional damping and torsional stiffness reduced the amplitudes. The physical mechanism causing the vibration was associated with reduction in aerodynamic pitch damping caused by stall, which led to large-amplitude torsional loads and high blade loads.

In an important sequel to this study, Ham and Young[35] conducted a study of stall flutter using a model rotor in hover. A single-degree-of-freedom limit-cycle torsional oscillation, with a frequency close to the natural torsional lag frequency of the blade, was found to occur at high collective pitch settings. The origin of this torsional motion was indicated by experimental study of chordwise pressure variation on the model rotor during the stable limit-cycle oscillation. Using a simple analysis, the relationships between the torsional motion and the effective damping in pitch in presence of stall are determined. Also the effect of reduced frequency on limit-cycle amplitudes was experimentally measured. The implication of the results obtained for the case of forward flight were also discussed, and a simple numerical method for approximating the boundary of stable pitch-torsional oscillation in forward flight was described and shown to produce good correlation with flight-test results.

B. Coupled Rotor-Fuselage Problems

In addition to isolated blade stability and response problems just discussed, one also encounters coupled rotor-fuselage problems as depicted in Fig. 8. Two types of problems were encountered. When the helicopter is on the ground, a mechanical instability couples in-plane blade motion with displacement of the axis of rotation caused by roll or pitch; this is usually denoted by the term "*ground resonance.* The second instability is in flight, and again it is caused by coupling between blade in-plane (lag) motion and body roll or pitch. This aeromechanical problem is usually denoted by the term *air resonance.* This terminology is unfortunate because neither phenomenon has anything to do with resonance.

Early in the development of rotorcraft, ground resonance and its avoidance were identified as major design issues. The first definitive study of ground resonance was carried out by Coleman and Feingold.[36] This report is a collection of the work done earlier by those two authors on two bladed rotors on isotropic and anisotropic supports, as well as rotors having three or more blades. The ground resonance represents coupling between a low-frequency lag mode (in the nonrotating frame) and a natural frequency of the structure

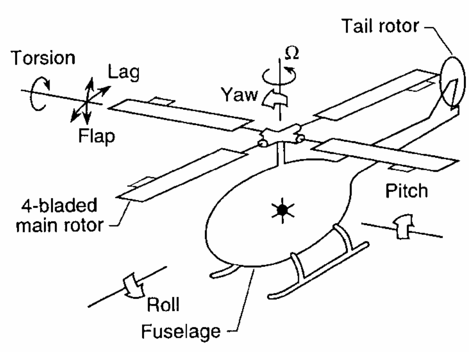

Fig. 8 Coupled rotor/fuselage system.

supporting the hub. This coupling produces lateral and longitudinal displacement of the rotor center of gravity from the center of rotation. Articulated rotors and hingeless rotors with lag frequency below 1/rev are susceptible to this instability. Ground resonance is very destructive. Although ground resonance was well understood in this time period, only a limited understanding of air resonance existed.

A valuable study conduced in the late 1960s[37] examined the air- and ground-resonance characteristics of a soft in-plane hingeless rotor system used on an experimental XH-51A helicopter built by Lockheed. The rotating fundamental lag frequency of soft-in-plane rotors is below 1/rev. The particular rotor considered in this study had a "matched stiffness" configuration, which eliminated part of the elastic coupling between the flap and lag degrees of freedom and causes the rotor to be more susceptible to the flap-lag type of instability. The paper has an excellent graphical description of the mechanism of ground/air resonance for soft-in-plane hingeless rotors, which occurs when the rotor rpm is is such that $\Omega - \omega_{ip}$ is close to a body natural frequency. In this case the center of gravity of the rotor disk is whirling about the center of rotation at an angular velocity ω_{ip}, as shown in Fig. 9. The $\Omega - \omega_{ip}$ curves relate body frequencies, as shown in Fig. 10. For the articulated rotor helicopter a critical body frequency for ground resonance coincides with the driving frequency when the rotor speed is at a value below the operating speed, which corresponds to the leftmost circle on the figure. The coincidence between the inclined dash–dot line with the double line marked (on the ground) indicates ground resonance. The soft-in-plane rotor tested and described in the paper can encounter ground resonance when it is at an rpm above the operating speed as shown by the intersection of the solid line and the double line denoted (on the ground); however, it can encounter air

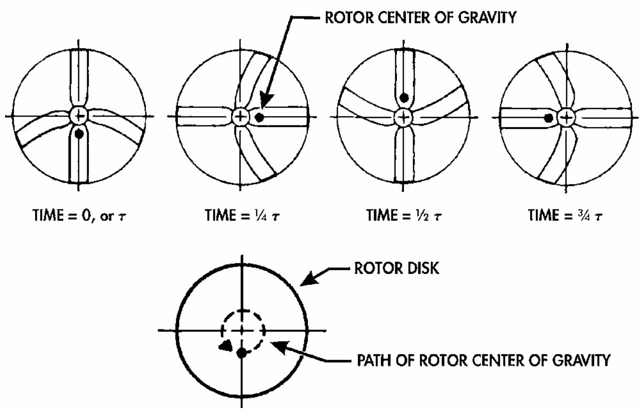

Fig. 9 Rotor-in-plane mode in the nonrotating system.

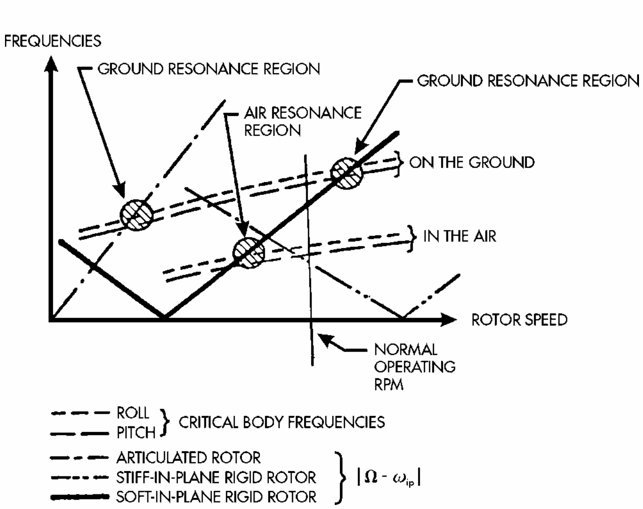

Fig. 10 Driving and body frequency relationships.

resonance if the rotor is slowed in flight, as indicated by the intersection of the solid line and the double line (in the air). As a result of the special construction of the rotor, both ground and air resonance were demonstrated experimentally, and analytical results were correlated with experimental data. However, the conclusions reached in this study were not definitive, mainly because of incomplete understanding of the appropriate structural dynamic modeling of hingeless rotor systems.

C. Summary of the State of the Art

To set the stage for a discussion of the next time period, a summary of the state of the art for the early time period is useful:

1) The pitch-flap and pitch-lag instabilities of articulated rotors were reasonably well understood, particularly for the case of hover. However, there was considerable confusion about the flap-lag type of instability. The unsteady aerodynamics was approximated by using Theodorsen- and Loewy-type unsteady aerodynamics.

2) For the case of forward flight, there was some understanding of the role of equations with periodic coefficients and its mathematical implications. However, there was little appreciation for effective numerical methods for dealing with such equations. There was also growing appreciation for the important role of retreating blade stall and stall flutter.

3) There was a good understanding of the ground-resonance problem, particularly for articulated rotors. The important role of lag dampers for preventing this problem was also appreciated.

However, despite the remarkable progress made and the successful design, engineering analysis, and production of a large number of successful helicopters, the state of the art had major deficiencies that needed to be overcome before additional progress could be made. These deficiencies are summarized here:

1) The flap-lag instability problem was not well understood.

2) There was only limited appreciation of systematic approaches to formulating and solving RWA problems.

3) Hingeless rotor aeroelastic behavior and air resonance were not understood.

4) There was no appreciation for the important role of structural dynamic models capable of representing coupled flap-lag-torsional dynamics in formulating RWA problems.

5) The role of geometric nonlinearities in RWA was not well understood.

6) Unsteady aerodynamic models, wake models, and dynamic stall models were not available.

7) Treatments of the true RWA problem, as represented by the coupled flap-lag-torsional problem in hover and forward flight, were not available.

III. Golden Age (1970–2000)

A. Overview of Principal Developments

This period was characterized by rising to the challenges posed by the unsolved problems summarized at the end of the preceding section. The accomplishments of this period were summarized in the various survey papers mentioned in the introductory portion of this paper. Before discussing the most important accomplishments in detail, it is useful to distinguish between two types: 1) accomplishments in modeling the aeroelastic behavior of rotor blade and coupled rotor-fuselage systems and 2) development of modern rotor systems, such as hingeless and bearingless, used on various rotorcraft being produced worldwide. Clearly, these two types of accomplishments are intertwined because modern rotor systems cannot be developed without certain aeroelastic modeling capability. Also, new modeling capabilities are being developed to meet the challenges of the hardware designer. Emphasis in this paper is on the most important developments in aeroelastic modeling techniques.

Some key developments in modeling of aeroelastic behavior that have occurred during this period are listed here: 1) recognition of the fundamental role of structural modeling and associated kinematic assumptions in the proper formulation of the RWA problem; 2) unsteady aerodynamics for attached and separated flow; 3) development of systematic tools for formulating and solving RWA problems; 4) understanding of the basic coupled flap-lag aeroelastic problem in hover and forward flight; 5) understanding of the coupled flap-lag-torsional problem in hover and forward flight; 6) understanding of air and ground resonance; 7) modeling of composite rotor blades; 8) modeling of hingeless, bearingless, and swept tip rotor blades; and 9) development of comprehensive analysis codes capable of modeling several RWA problems.

A detailed description of all of these items within the framework of a single paper is quite difficult, and therefore one has to be selective so as to limit the paper to a reasonable length.

B. Role of Structural Modeling

Initially, structural models for isotropic rotor blades were linear,[29,38] and thus no distinction was made between the deformed and undeformed blade configurations. The aeroelastic formulations developed in the late 1960s were all based on the Houbolt and Brooks equations.[29] In the late 1960s and early 1970s it was recognized that geometrical nonlinearities caused by moderate deflections needed to be incorporated in the aeroelastic operators associated with the rotary-wing aeroelastic problem. The distinction between the undeformed and deformed blade geometries also produces nonlinear terms that have to be included in the inertia and aerodynamic operators. Moderate-deflection beam theories capable of representing the coupled flap-lag-torsional dynamics of rotor blades were developed primarily between 1970–1980, and during the next decade large deflection theories were derived. The inception of moderate deflection theories can be found in two dissertations that were published in the same year.[39,40] An integral part of moderate deflection theories was ordering schemes, which allowed one to neglect higher-order terms in the structural, aerodynamic, and inertia operators associated with the aeroelastic problem. Subsequently, the equations evolved, and more careful derivation of the structural part resulted in equations that have formed the basis of numerous aeroelastic studies.[41,42]

The source and structure of the geometrically nonlinear terms associated with structural rotations are conveniently illustrated by a transformation between the triad of unit vectors describing the deformed and undeformed state of a hingeless blade, as shown in Fig. 11. Only four independent functions (three displacement variables and one rotation) are needed for the exact form of this transformation because of a constraint that the plane in which the vectors $\hat{e}_y$ and $\hat{e}_z$ lie remains normal to the deformed beam elastic axis. If these vectors are, in turn, assumed to lie in the deformed beam cross section, then this constraint becomes analogous to the Euler–Bernoulli hypothesis for a large-deformation theory. Such a transformation, based on the assumption of small strains and finite rotations (associated with the twist angle and bending slopes), has the following mathematical form:

$$\begin{Bmatrix} \hat{e}'_x \\ \hat{e}'_y \\ \hat{e}'_z \end{Bmatrix} = [S] \begin{Bmatrix} \hat{e}_x \\ \hat{e}_y \\ \hat{e}_z \end{Bmatrix} \tag{5}$$

where the elements of the transformation matrix $[S]$ determine the accuracy or order of the theory. A typical transformation where terms up to the third order are accounted for is given here:

$$S_{11} = 1 - \tfrac{1}{2}\left(v_{,x}^2 + w_{,x}^2\right), \qquad S_{12} = v_{,x}, \qquad S_{13} = w_{,x}$$

$$S_{21} = -\left(v_{,x} - \phi w_{,x} + \tfrac{1}{2} v_{,x} w_{,x}^2\right), \qquad S_{22} = 1 - \tfrac{1}{2} v_{,x}^2 - \phi v_{,x} w_{,x}$$

$$S_{23} = \phi - \tfrac{1}{2} w_{,x}^2 \phi, \qquad S_{31} = -\left(w_{,x} - \phi v_{,x}^2 - \tfrac{1}{2} v_{,x}^2 w_{,x}\right)$$

$$S_{32} = -\left(\phi + v_{,x} w_{,x} - \tfrac{1}{2} v_{,x}^2 \phi\right), \qquad S_{33} = 1 - \tfrac{1}{2} w_{,x}^2 \tag{6}$$

Such a transformation can be assumed to imply the existence of an ordering scheme in which third-order terms, in terms of blade

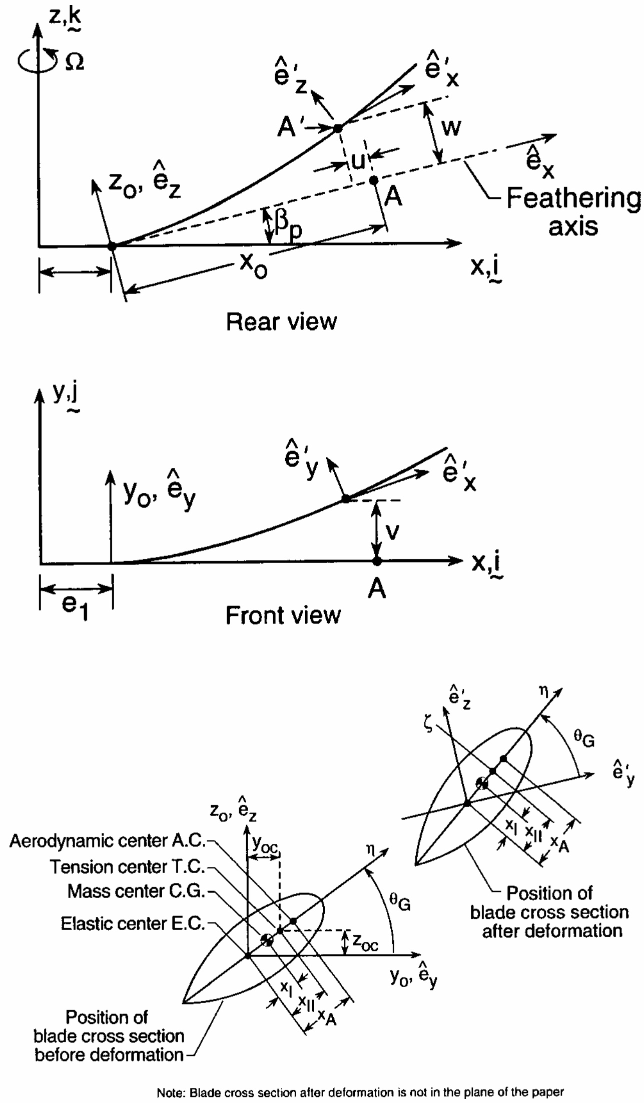

Fig. 11 Geometry of the blade elastic axis before and after deformation (top) and blade cross-sectional geometry before and after deformation (bottom).

slopes, are neglected. Such an ordering scheme implies

$$\mathcal{O}(1) + \mathcal{O}(\epsilon^3) \cong \mathcal{O}(1) \tag{7}$$

where blade slopes are assumed to be moderate and of magnitude ϵ, that is, $0.10 \leq \epsilon \leq 0.20$. Use of a less accurate ordering scheme

$$\mathcal{O}(1) + \mathcal{O}(\epsilon^2) \cong \mathcal{O}(1) \tag{8}$$

will lead to the neglect of the third-order terms in Eqs. (6). A word of caution is in order at this point. To allow for the treatment of applied moments, the virtual rotation must be obtained as a function of the deformation variables. The variation must be taken prior to the neglect of the third-order terms; otherwise, the expressions for virtual rotation will be incorrect (see Refs. 43 and 44 for more detail on this point). Transformations of the form of Eq. (5) have been used as the basis for moderate-deflection beam theories, which are suitable for the aeroelastic stability and response analysis of isotropic hingeless and bearingless rotor blades. Once a transformation represented by Eq. (5) is available, it is used to derive the inertia and aerodynamic loads acting on the blade. Thus, these terms permeate through the entire set of equations of motion describing the dynamics of the blade.

Consider as an example the treatment of the coupled flap-lag-torsional dynamics of an isolated blade in forward flight. For this case the ordering scheme would be based on the order of magnitude assumptions given here:

$$w_{,x} = v_{,x} = \phi = \mathcal{O}(\epsilon)$$

$$\frac{e_1}{R} = \frac{b}{R} = \beta_p = \bar{\lambda} = \lambda_{1s} = \lambda_{1c} = \frac{w}{R} = \frac{v}{R} = \mathcal{O}(\epsilon)$$

$$\theta = \theta_{1c} = \theta_{1s} = \mathcal{O}(1)$$

$$u = x_I/R = x_A/R = \mathcal{O}(\epsilon^2), \qquad C_{d0}/a = \mathcal{O}\left(\epsilon^{\frac{3}{2}}\right)$$

$$x/l = \frac{\partial}{\partial x} = \frac{\partial}{\partial \psi} = \mu = \mathcal{O}(1) \tag{9}$$

Application of such an ordering scheme leads to the neglect of numerous higher-order terms. Furthermore, modern computer packages capable of algebraic manipulation, such as Mathematica®, can be used together with an ordering scheme to generate equations with a desired level of accuracy.[45]

Finally, such a scheme is based on common sense and experience with practical blade configurations. Thus, it should be applied with a certain degree of flexibility.

Structural models for moderate, as well as large deflection beam (or blade) theories, have been often validated by correlating them with experimental data obtained in a static experiment conducted at Princeton.[46]

The development of moderate deflection beam theories was followed by structural models that use only the smallness of the extensional strain; otherwise, the analysis allows for arbitrarily large deflections and rotations. This approach completely eliminates the need for an ordering scheme. This type of model is more consistent and mathematically more elegant than blade models based on ordering schemes. References 47–50 are representative of the first studies that have established this more accurate approach.

When the strain is assumed to be small, two developments are feasible, depending on the representation of the cross-sectional deformation. Consider the rotation of the deformed beam sectional frame, which is assumed to be arbitrarily large; this is denoted as global rotation. Furthermore, consider the rotation of a material element at some point in the cross section caused by cross-sectional deformation. This so-called local rotation is relative to the deformed beam sectional frame. The simpler development assumes that local rotation is of the order of the strain, whereas the more general one assumes that the local rotation is of the order of the square root of the strain. In either case the beam deformation can be expressed in terms of six generalized strain measures: the extension of the reference axis, two shear strains at the reference axis, the elastic twist, and two elastic bending measures. Because of the presence of shear-strain measures, three independent orientation variables must be allowed, as in Ref. 47. That is, it is not possible to express two of the orientation variables in terms of the derivatives of the three displacement variables. Also, although not necessary for static and low-frequency analysis of composite rotor blades, the presence of shear strain in these developments improves their accuracy in applications to composite rotor blade analysis when transverse bending modes higher than the first are involved.

C. Unsteady Aerodynamics for Attached and Separated Flow

Accurate modeling of the unsteady aerodynamic loads required for aeroelastic stability and response calculation continues to be one of the major challenges facing both the analyst and the designer. The combination of the blade advancing and rotational speed is a formidable source of complexity in the flowfield surrounding the rotor. At large values of the advance ratio, the aerodynamic flowfield around the blade undergoes such variations that there are problems of transonic flow, with the shock waves on the advancing blade tip, problems of flow reversal (reversed flow region) and low-speed, unsteady stall on the retreating blade, and problems caused by high blade-sweep angle for various azimuthal locations. Modern swept and curved-tip blade geometries further complicate this problem. Furthermore, the time-varying geometry of the wake, which is an important source of unsteady loads, vibration and noise, is an excruciatingly complex problem that is an order of magnitude more complicated that the wake geometry of fixed wings.

When dealing with the unsteady aerodynamic problem, one can make a wide array of assumptions, which lead to diverse models, starting with simple and computationally efficient models and culminating in models, which are capable of simulating the more intricate details of the unsteady flow. A detailed description of unsteady aerodynamic models for rotary-wing applications has been presented in books,[17,19] as well as a couple of review papers.[51,52]

1. *Attached-Flow Unsteady Aerodynamics*

From the first part of this paper, it is evident that the unsteady aerodynamic models available were limited to two-dimensional incompressible theories such as Theodorsen, Greenberg,[33] and Loewy.[32] Because of the low reduced frequency associated with RWA problems, unsteady aerodynamic effects have been found to be of less than critical importance. Furthermore, because of its wake structure Theodorsen's theory is not suitable for rotary-wing application, whereas Loewy's theory is limited to lightly loaded rotors.

It is also important to recognize that both are frequency domain theories, which are not suitable for forward flight, where the equations of dynamic equilibrium have periodic coefficients. For convenient mathematical treatment of equations with periodic coefficients, time-domain theories are required. Therefore, Greenberg's theory with appropriate modifications[53–56] has been often used in RWA, with the assumption that the aerodynamics are quasisteady, $C(k) = 1$. For this case the theory was also used in forward flight.

Loewy's theory has been extended to include compressibility effects. However, these theories have been rarely used in coupled flap-lag-torsional analysis in hover.[53]

Frequency-domain theories have a significant deficiency when being applied to aeroelastic stability calculations because the assumption of simple harmonic motion upon which they are based implies that they are strictly only valid at the stability boundary. Thus, they provide no information on system damping before or after the flutter condition is reached, and standard stability analyses based on conventional eigenanalysis, such as the root locus method, cannot be used. Furthermore, as indicated before, these are not suitable for rotary-wing aeroelastic analyses in forward flight or applications where the transient response of the aeroelastic system is required. Thus, there is a need for unsteady aerodynamic theories that are capable of modeling unsteady aerodynamic loads in the time domain for finite-time arbitrary motion of an airfoil, representing the cross section of an oscillating rotor blade. The term "arbitrary motion" is used here to denote growing or decaying oscillations with a certain frequency. A number of such theories were developed, and Refs. 5 and 57 contain a unified description of such theories.

Time-domain airfoil theories are extensions of previous frequency-domain theories, using an approach developed by Edwards[58] to extend Theodorsen's theory to the time domain. Time-domain versions of Greenberg's theory can be found in Ref. 59, and a time-domain version of Loewy's theory was presented in Ref. 60.

A particularly useful time-domain theory, which has been used frequently in rotary-wing aeromechanical applications, is the dynamic inflow model, which was developed and used first at the beginning of the 1980s.[61–63] The mathematical form of the dynamic inflow model in both hover and forward flight clearly indicates that it is an arbitrary motion, time-domain theory. The most widely used version of dynamic inflow is that developed by Pit and Peters,[63] which is suitable for both hover and forward flight. The model represents unsteady global wake effects in a simple form and is applicable to the entire rotor. The assumption in this theory is that, for relatively low frequencies, actuator disk theory is valid for both steady and unsteady conditions. Therefore, dynamic inflow is essentially a low-frequency approximation to the unsteady aerodynamics of the rotor. The total induced velocity on the rotor disk is assumed to consist of a steady inflow λ_0 (for trim loadings) and a perturbational inflow, denoted $\delta\lambda$, as a result of transient loadings. The total inflow is expressed as:

$$\lambda = \lambda_0 + \delta\lambda \tag{10}$$

where $\delta\lambda$ is assumed to be given by

$$\delta\lambda = \lambda_1 + \lambda_{1c}\left(\frac{r}{R}\right)\cos\psi + \lambda_{1s}\left(\frac{r}{R}\right)\sin\psi \tag{11}$$

in which the inflow variables λ_1, λ_{1c}, and λ_{1s} are related to the perturbational thrust coefficient, roll and pitch-moment coefficients acting on the rotor through the following relation:

$$[M]\begin{Bmatrix}\dot{\lambda}_1\\ \dot{\lambda}_{1c}\\ \dot{\lambda}_{1s}\end{Bmatrix} + [L]^{-1}\begin{Bmatrix}\lambda_1\\ \lambda_{1c}\\ \lambda_{1s}\end{Bmatrix} = \begin{Bmatrix}C_T\\ -C_{My}\\ C_{Mx}\end{Bmatrix}_{\text{P.A.}} \tag{12}$$

P.A. stands for perturbational aerodynamics. The elements of $[M]$ and $[L]^{-1}$ can be obtained either theoretically, by using momentum theory,[63] or experimentally. Dynamic inflow models have been particularly useful for coupled rotor-fuselage aeromechanical problems in both hover and forward flight, and they have been used for isolated rotor stability analyses.[64]

Subsequently, the concept of dynamic inflow has led to the development of a complete unsteady aerodynamic model applicable to RWA.[65,66] In this theory the induced flow on the rotor disk is expanded in Fourier coefficients (azimuthally) and spatial polynomials (radially). The coefficients of these expansion terms are shown to obey a closed-form set of ordinary differential equations with blade loading (from any source) as the forcing functions. The obvious advantage of such an approach is that the resultant equations can be used for arbitrary motions in the time domain (time-marching or Floquet), in the frequency domain (harmonic balance), or in the eigenvalue domain (conventional stability analysis) to any degree of resolution as dictated by the application.

This theory is derived from the linear potential equations with a skewed cylindrical wake. Wake contraction can also be modeled. For hover the results of this theory agree with Loewy's model. A convenient feature of this theory is that it can be easily coupled with Floquet solution of the equations of motion in forward flight. A shortcoming of the theory is that it cannot model the important effect of blade-vortex interaction, which can be captured only by free wake models.

2. Separated-Flow Unsteady Aerodynamics—Dynamic Stall

Dynamic stall is a strong nonlinear unsteady aerodynamic effect associated with flow separation and reattachment, which plays a major role in aeroelastic stability and response calculations. Good descriptions of dynamic stall can be found in Refs. 17 and 19. In the early years dynamic stall was not well understood, and models that would allow one to incorporate dynamic stall in an aeroelastic analysis were not available.

Dynamic stall is associated with the retreating blade and borders on the reversed flow region, as shown in Fig. 12. For such conditions the angle of attack of the blade cross section can be very large. Although the torsional response of the blade is relatively low under normal conditions, at the flight envelope boundary, where dynamic stall effects are pronounced, large transient-torsional excursion can be excited, accompanied by low negative damping in pitch. This, in turn, generates excessive control and blade vibratory loads, which impose speed and load limitations on the rotor system as a whole. It can also cause stall flutter. Because of its importance, dynamic stall has been the subject of a large number of studies, which have led to a good physical understanding of this complex aerodynamic effect. Some of the earlier work on this topic was done by Ham,[67] and subsequent experimental and analytical work of Carr[68] and McCroskey and his associates[69] has led to improved physical understanding of this phenomenon. Attempts at simulation of dynamic stall using computational fluid dynamics have not been successful in reproducing the quantitative characteristics at operational Reynolds numbers. The need to incorporate the important dynamic-stall effects in rotary-wing aeroelastic stability and response calculations has led to the development of semi-empirical dynamic-stall models that capture the most important features of dynamic stall with

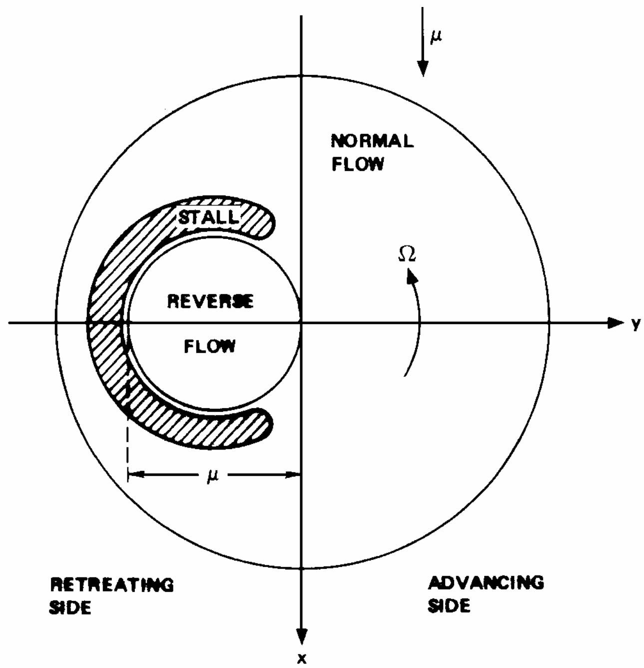

Fig. 12 Schematic illustration of reversed flow region and dynamic-stall region.

reasonable accuracy. Semi-empirical models can reproduce the hysteretic lift, moment and drag curves for a given airfoil quite accurately. These models have a number of common features. They are intended to incorporate two-dimensional airfoil unsteady aerodynamic effects in analytical studies in the time domain, and they are suitable for stepwise numerical integration in time. All models are empirical, and various free parameters in the model are determined by fitting the theory to experimental data obtained from oscillating airfoil tests.

Several dynamic-stall models have been developed. However, only two have withstood the test of time and are in widespread use currently. These are the ONERA and the Leishman–Beddoes dynamic-stall models. Both distinguish between two principal flow regions: the attached, and the separated-flow regions.

The ONERA model developed by Dat,[51] Dat et al.,[70] and Tran and Petot[71] is based on the time-domain representation of the airfoil section operating before, during, and in the poststall regime while it performs essentially arbitrary motions. The model utilizes the properties of differential equations to simulate the different effects that can be identified on an oscillating airfoil, such as pseudoelastic, viscous and inertial effects, and the effect of the flow time history. The theory also recognizes that, in the linear range of airfoil motions, Theodorsen's lift-deficiency function represents the aerodynamic transfer function for the airfoil, relating the downwash velocity at the three-quarter chord to circulatory lift. Furthermore, the theory is based on approximating the aerodynamic transfer function by rational functions. In the nonlinear range the model consists of a system of differential equations containing unsteady linear terms whose coefficients are functions of the angle of attack and steady-flow nonlinear terms.

The ONERA model has been modified and improved by Rogers[72] and Peters.[73] These changes have produced a modified theory, which in the attached-flow region is consistent with classical unsteady aerodynamics and in which circulation has been introduced as a new dependent variable. The ONERA model contains an approximate correction for compressibility and no correction for the effect of sweep. The most recent version of this model was documented by Petot.[74] The coefficients in the equations of this model are determined by parameter identification from experimental measurements on oscillating airfoils. The model requires 22 empirical coefficients. Figure 13 shows typical hysteretic lift and moment coefficients computed with the ONERA dynamic-stall model for a NACA 0012

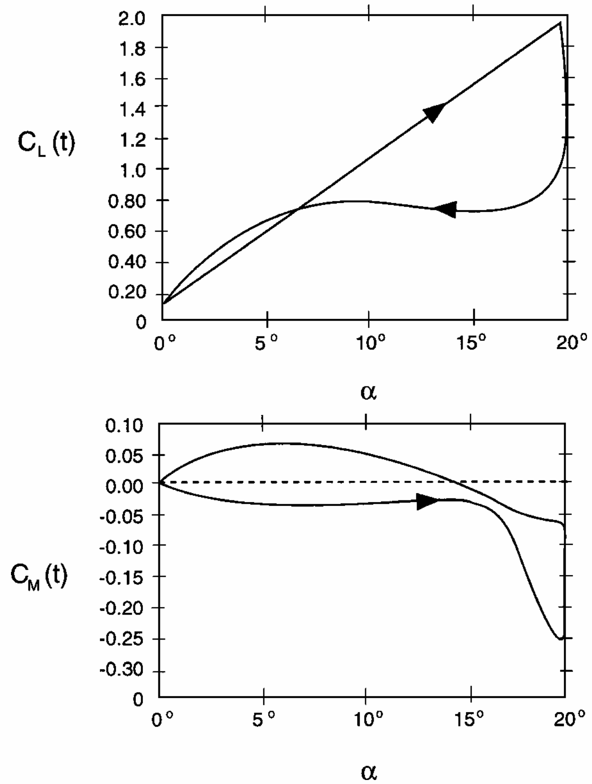

Fig. 13 Typical hysteretic lift and moment coefficients computed with the ONERA dynamic-stall model.

airfoil at $M = 0.379$, $k = 0.075$, and a time-varying angle of attack $\alpha = 10.3$ deg $+8.1 \sin \omega t$.

The Leishman–Beddoes model was developed originally by Beddoes in the mid-1970s.[75,76] Subsequently, it was extended by Leishman,[77–79] and it has become a comprehensive and mature model. The model is capable of representing the unsteady lift, pitching moment, and drag characteristics of an airfoil undergoing dynamic stall. This model consists of three distinct components: 1) an attached-flow model for the unsteady linear airloads, 2) a separated-flow model for the nonlinear airloads, and 3) a dynamic-stall model for the leading-edge vortex-induced airloads. The model contains a rigorous representation of compressibility in the attached-flow part of the model, using compressible indicial response functions. The treatment of nonlinear aerodynamic effects associated with separated flows are derived from the Kirchoff–Helmholtz model to define an effective separation point that can be generalized empirically. The model uses relatively few empirical constants, with all but four derived from static airfoil data.

3. Wake Models

The description of aerodynamic loading is incomplete without mentioning wake models. A detailed description of wake models and their historical development is outside the scope of this paper, but can be found in Chapter 10 of Ref. 19. Accurate modeling of the wake and, in particular, free-wake models plays a critical role in aeroelastic response and blade vibratory load calculations. However, it appears that accurate modeling of the wake is less important for aeroelastic stability analyses.

D. Development of Systematic Methods for Formulating and Solving Rotary-Wing Aeroelastic Problems

1. Formulation of Equations of Motion

Formulation of complete aeroelastic equations of motion requires a combination of structural, aerodynamic, and inertia terms. It is in this area that very significant advances have been made compared to the early period when equations of motion were formulated using ad hoc methods augmented by good physical insight. The structural and aerodynamic ingredients have been described in the preceding sections. The inertia loads are obtained in a straightforward manner by using D'Alambert's principle and combining it with Newton's Second Law. The transformation between the undeformed and deformed states represented by Eq. (5) determines the position vector of a mass point in the deforming blade, and the acceleration vector is given by using vector mechanics:

$$\boldsymbol{a} = \ddot{\boldsymbol{R}} + 2\boldsymbol{\Omega} \times \boldsymbol{R} + \dot{\boldsymbol{\Omega}} \times \boldsymbol{R} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{R}) \tag{13}$$

Although the derivation of the inertia terms is conceptually simple, the practical implementation can be tedious from an algebraic point of view, particularly when one is interested in coupled rotor fuselage aeromechnical problems. In the 1970s and early 1980s equations of motion used to be derived manually leading to long and algebraically cumbersome expressions. Examples illustrating the complexity of the equations for isolated blade aeroelastic problems in forward flight[80,81] or coupled rotor-fuselage problems[82–84] clearly indicate the tedious nature of such tasks, even when ordering schemes are used. Furthermore, when advanced aerodynamics such as dynamic-stall or wake models are used[82] it is impossible to obtain explicit equations of motion.

The equations are partial differential equations, and their solution requires discretization to eliminate the spatial dependence. Discretization and the solution of the equation requires further algebraic effort. Finally, the finite element method, which was first used for a rotary-wing aeroelastic problem in 1980 (Ref. 85) and has become the most effective spatial discretization and solution method currently used in RWA, tends to obscure the precise boundaries between problem formulation and solution.

Since the early 1970s, two distinct approaches for formulating isolated blade or coupled rotor-fuselage equations of motion have emerged. The first approach is usually denoted the explicit approach because it leads to a set of detailed aeroelastic equations of motion in which all of the terms (aerodynamic, structural, and inertial) appear in explicit form. Explicit equations are usually derived using ordering schemes to neglect higher-order terms in a systematic manner. The outcome of this process consists of a set of nonlinear partial differential equations in the space and time domain. These equations can also contain integral expressions caused by centrifugal and other terms. An alternative approach can be denoted as the implicit approach. In this approach the detailed expressions of the aeroelastic equations of motion are avoided; instead, the aerodynamic, inertia, and structural operators are usually generated in matrix form inside the computer. When this approach is used, the boundaries between the formulation and solution process, particularly in spatial discretization, tend to be blurred. When the implicit approach is used, ordering schemes are no longer required. Furthermore, the implicit approach frequently mandates iterative solutions. For convenience and clarity the implementation of these two approaches will be discussed by describing their application to two separate classes of problems, namely, isolated-blade problems and coupled rotor-fuselage problems.

Isolated-blade case. A good example of explicit formulation of equations of motion for the case of hover can be found in Refs. 41 and 54. Explicit formulations for forward flight can be found in Refs. 80 and 81. The algebraic task for deriving such equations was too cumbersome, and, therefore, with increases in computer power these tasks have been relegated to the computer. One of the first derivations of a set of coupled flap-lag-torsional equations of motion for a hingeless rotor blade in forward flight, using a special purpose symbolic processor written in FORTRAN, was presented by Reddy and Warmbrodt.[86]

In the mid-1980s LISP workstations utilizing the MACSYMA symbolic manipulative package became commercially available and were used to derive coupled flap-lag-torsional equations for a hingeless blade in hover, including terms up to the third order.[45,87,88] By the early 1990s regular workstations could be used in conjunction with MACSYMA to obtain explicit equations for hingeless rotor blades in forward flight in a routine manner.[89,90]

Another approach for avoiding the algebraic derivation associated with the formulation of the RWA problem is to use the finite element approach. For this approach the equations of motion are generated in a numerical form, as part of the solution process. The approach can be used to obtain either explicit or implicit formulations and solutions. This approach will be described later in this paper because the finite element formulation is strongly linked to the solution methodology.

Coupled rotor-fuselage case. Formulation of coupled rotor-fuselage equations for a typical configuration, like that shown in Fig. 8, has similarities to the isolated-blade problem, although a number of substantial differences do exist: 1) the equations are more complicated because of numerous additional terms associated with the fuselage rigid-body degrees of freedom, which contribute to the complexity of the inertia and aerodynamic loads; 2) if ordering schemes are used, combined with an explicit formulation, a modified form of the ordering scheme has to be used to restrict the equations to a manageable size; 3) rotor-fuselage coupling has to be performed in a careful and systematic manner; and 4) when the fuselage itself is also considered as a flexible body, a further complication in problem formulation emerges. Again, like in the isolated blade case, both explicit and implicit formulations of the coupled rotor-fuselage problem are available.

The early derivations, developed in the late 1970s, for this class of problems were explicit, usually done manually and resulted in lengthy equations. References 91 and 92 good examples of coupled rotor-fuselage equations in hover, whereas Ref. 84 is representative of typical equations for forward flight. In the same period Johnson[93] has developed a coupled rotor-fuselage model suitable for forward flight using an implicit approach. In the studies just mentioned, the fuselage was assumed to be rigid. In the mid-1980s there was a need to obtain coupled rotor-fuselage equations capable of modeling single- and twin-rotor configurations in hover and forward flight, and thus these were derived and solved in Refs. 83 and 94. The fuselage was modeled as a flexible beam with bending and torsional degrees of freedom. This study represents one of the most complicated explicit sets of equations derived manually. Three years later they were rederived using MACSYMA and found to be error free.

Obviously, this class of problems is an ideal candidate for symbolic derivation of equations motion. The first explicit formulations based on symbolic manipulation were carried out in Ref. 95.

An interesting implicit approach, based on a hybrid finite element combined with a primitive multibody formulation, was used in the GRASP program.[96] The required matrix elements were generated by numerical evaluation of hierarchical expressions in the code. As one of the first multi-flexible-body approaches applicable to rotorcraft, GRASP possessed a lot of modeling flexibility. Unfortunately, however, the only version released until the time development was halted was limited to the hovering flight condition.

2. *Methods of Solution*

The solution of rotary-wing aeroelastic stability and response problems is usually carried out in two stages. The first stage consists of the spatial discretization of the equations of motion followed by a solution in the time domain. In the second stage, namely, the time-domain solution, two different approaches are possible; one can solve the equation in a blade-fixed, rotating coordinate system or in a hub-fixed, nonrotating coordinate system. One also needs to distinguish between solutions for hover and those for forward flight.

Spatial discretization. The first step in the solution of rotary-wing aeroelastic stability or response problems is elimination of the spatial dependence in the nonlinear partial differential equations (or appropriate energy expressions), which describe the system. Application of suitable discretization methods will yield a set of coupled, nonlinear, ordinary differential equations in the time domain. Three approaches for spatial discretization have been used: 1) spatial discretization based on global methods, 2) spatial discretization based on the finite element method, and 3) spatial discretization based on matrix method. The third approach is mentioned only for historical reasons. This approach has not withstood the test of time and will not be discussed here, but information on this approach can be found in Ref. 9.

During the 1970s, the preferred discretization methods were global, such as the well-known Rayleigh–Ritz or Galerkin methods, as shown in Refs. 3, 54, 86, and 97 based on free-vibration modes of the rotating blade. For hover, both coupled and uncoupled modes have been used, where the coupling of modes is caused by the collective pitch setting. For forward flight the use of uncoupled modes is more convenient because cyclic pitch introduces time-varying coupling. Discretization based on global modes is cumbersome and is best handled by symbolic computation or numerical implementation using Gaussian quadrature.

Since 1980, the finite element method has emerged as the most versatile spatial discretization method. In addition to eliminating the algebraic manipulative labor required for the solution of the problem, it also serves as the basis for the implicit formulations discussed in the preceding section. Furthermore, the finite element method is ideally suited for modeling composite rotor blades and complicated redundant structural systems such as encountered in bearingless rotors. For rotary-wing aeroelastic problems two approaches have been used: 1) weighted residual Galerkin-type finite element methods and 2) local Rayleigh–Ritz finite element method using conventional as well as higher-order elements. Recognizing that the rotary-wing aeroelastic problem is geometrically nonlinear, it should be emphasized that finite element formulation for this class of problems is more intricate than its fixed-wing counterpart.

The first finite element treatment of the rotary-wing aeroelastic problem in hover and forward flight, using a Galerkin-type weighted residual finite-element method, can be found in Refs. 85, 98, and 99. First, the coupled flap-lag problem was treated,[85,98] and subsequently the coupled flap-lag-torsional problem was formulated.[99] The geometry for this problem is shown in Fig. 14. The bending degrees of freedom were interpolated using cubic (or Hermite) interpolation, whereas quadratic interpolation was used for the torsional degree of freedom (not shown). In Refs. 85, 98, and 99 an explicit formulation was used; however, later the same method was used in an implicit formulation to solve the coupled flap-lag-torsional problem of straight and swept-tip hingeless rotor blades in hover and forward flight.[100,101]

The local Rayleigh–Ritz type finite element method was first used by Sivaneri and Chopra[102,103] to study the behavior of hingeless[102] and bearingless[103] rotors. Again, the bending degrees of freedom were treated using Hermite interpolation, and torsion was treated by

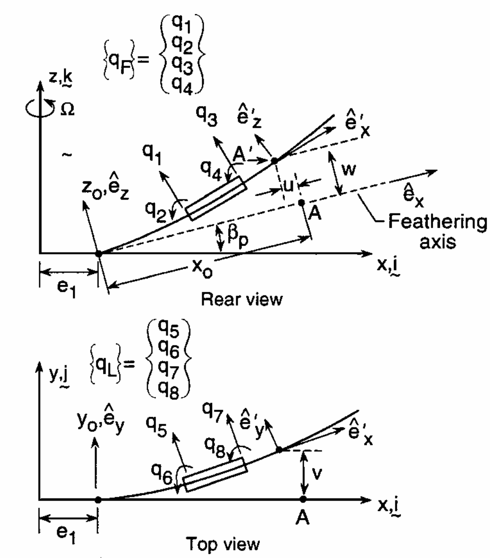

Fig. 14 Geometry of the elastic axis of the hingeless deformed blade and schematic representation of the finite element model.

linear interpolation.[102] Ritz-type higher-order finite elements were combined with an implicit formulation and used in the GRASP program to solve aeroelastic problems in hover.[96]

Time-domain solution of the equations. After spatial discretization the equations of motion are reduced to nonlinear ordinary differential form. In forward flight these equations have periodic coefficients. The mathematical structure of these general equations can be written in the following symbolic form[4]:

$$[M]\{\ddot{\boldsymbol{q}}\} + [C(\psi)]\{\dot{\boldsymbol{q}}\} + [K(\psi)]\{q\} = \{F_{\mathrm{NL}}(\psi, \boldsymbol{q}, \dot{\boldsymbol{q}})\} \tag{14}$$

where it is understood that the matrices $[M]$ and $[C(\psi)]$ contain both aerodynamic and inertial contributions, whereas the matrix $[K(\psi)]$ contains aerodynamic, inertial, as well as structural contributions. All nonlinear effects are combined in a vector $\{F_{NL}(\psi, \boldsymbol{q}, \dot{\boldsymbol{q}})\}$.

When time-domain unsteady aerodynamics such as Eqs. (12) are used, these equations have to be appended to Eqs. (14) and solved jointly.[61] When discussing methods of solution for Eq. (14), it is convenient to consider hover and forward flight separately. It is also useful to distinguish between isolated-blade and coupled-rotor-fuselage analyses.

Consider the first isolated blade case in hover. For this case Eq. (14) has constant coefficients. Linearizing the equations about a nonlinear equilibrium position gives a good approximation to aeroelastic stability boundaries.[3,4,39,54,85] Thus, it is common practice to write perturbation equations that are linearized about the nonlinear static equilibrium position. This equilibrium position is obtained from the solution of a system of a system of nonlinear algebraic equations. These equations are usually solved by Newton–Raphson iteration.[3,4,104] Lack of convergence can be indicative of divergence. Stability boundaries are obtained from solving the standard eigenvalue problem for the linearized system. The real part of the eigenvalues determines the aeroelastic stability boundaries of the blade. This approach also predicts reliable stability boundaries in the presence of static stall.[105] This basic approach has evolved during the 1970s and has been used ever since then without any significant changes.

Consider next the isolated blade in forward flight, and assume that dynamic-stall effects are neglected. The aeroelastic stability and response problem is based again upon Eq. (14). Reliable solutions for stability can be obtained by linearizing the nonlinear equations of motion about an appropriate equilibrium position.[82,86,97,106] In forward flight the appropriate equilibrium position is a periodic solution of Eq. (14). Calculation of the time-dependent periodic equilibrium position, representing the blade response solution, is inherently coupled with the trim state, or flight mechanics, of the complete helicopter in forward flight. The degree of sophistication with which this coupling is accomplished can affect the accuracy of the aeroelastic analysis. The trim state of a typical helicopter is depicted in Fig. 15.

Two different trim procedures[4,97,106] have been used: 1) propulsive trim, which simulates actual forward flight conditions where horizontal and vertical force equilibrium is maintained and zero pitching and rolling moments are enforced, and 2) moment or wind tunnel trim, which simulates conditions under which the rotor would be tested in a wind tunnel. Horizontal and vertical force equilibrium is not enforced because the helicopter is mounted on a supporting structure. Therefore, only the requirement of zero moments on the rotor is imposed.

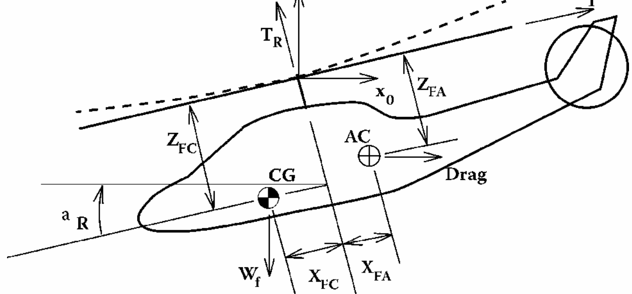

Fig. 15 Geometry showing trimmed aircraft in propulsive trim.

Initially, the trim procedures used in the mid 1970s to mid 1980s coupled trim only partially to the aeroelastic analysis, and the first time trim requirements on all three flap, lag, and torsional degrees of freedom were imposed was in Ref. 86. However, from the late 1980s onward coupled trim/aeroelastic analyses have been used in a routine manner.

The most widely used method for solving the forward flight problem is based on the direct solution of Eq. (14) in the rotating system, by using the theory of differential equations with periodic coefficients. This approach is facilitated by rewriting Eq. (14) in first-order, state-variable form

$$\{\dot{y}(\psi)\} = \{z(\psi)\} + [L(\psi)]\{y(\psi)\} + \{N(\psi, \boldsymbol{y}, \dot{\boldsymbol{y}})\} \tag{15}$$

where $\{z(\psi)\}$ and $[L(\psi)]$ are periodic matrices with common period 2π, and $\{N(\psi, \boldsymbol{y}, \dot{\boldsymbol{y}})\}$ represents the nonlinear contributions, and

$$\{y(\psi)\} = \begin{Bmatrix} \dot{\boldsymbol{q}}(\psi) \\ \cdots\cdots \\ \boldsymbol{q}(\psi) \end{Bmatrix} \tag{16}$$

It is evident from Eq. (15) that the aeroelastic stability and response problem are coupled. Therefore, solutions have to be obtained in two stages.[4,97] First the nonlinear time-dependent equilibrium position of the blade is obtained, and next the equations are linearized about the time-dependent equilibrium position by writing perturbation equation about the periodic nonlinear equilibrium. This second stage yields a linear periodic system for which blade stability can be obtained using Floquet theory. In the early years there was considerable confusion regarding the treatment of equations with periodic coefficients. One of the first papers to point out that calculation of the Floquet transition matrix at the end of one period, using numerical integration, is an effective way for dealing with flapping dynamics of a rotor in forward flight was published in the early 1970s (Ref. 107). A couple of years later it was shown[108] that the transition matrix at the end of a period is a key ingredient for examining aeroelastic stability in forward flight. An effective numerical technique for calculating the transition matrix at the end of one period was also presented.[108] This approach was generalized in a later paper[109] that presented very efficient numerical techniques for dealing with periodic systems by using Floquet theory. Later, the methods developed for the stability of linear periodic systems were extended to obtain the response for the linear case, as well as the nonlinear case using quasilinearization.[97] The importance of the treatment of equations with periodic systems has played an important role in RWA in forward flight and numerous papers on this topic have been written since the 1970s, including several survey papers.[110,111] Three distinct methods for calculating the nonlinear equilibrium position associated with Eq. (15), about which the perturbation equations are linearized, have emerged. These are quasilinearization,[97] periodic shooting,[112] and the finite element method applied in the time domain.[113]

An alternative approach to the solution of Eq. (14) is based on the introduction of the multiblade coordinate transformation, which transforms the variables $\{q\}$ from the rotating, blade-fixed reference frame, to a nonrotating, hub-fixed reference frame.[66,110,114,115] Following Hohenemser and Yin,[115] these coordinates are frequently used in rotor dynamics. Use of such a transformation implies that the blade degrees of freedom in the blade-fixed system are replaced by a set of nonrotating coordinates that describe the motions of the hub plane and its deformation in a hub-fixed system. In a symbolic form the multiblade coordinate transformation is written as

$$\{q\} = [B(\psi)]\{X\} \tag{17}$$

where $\{q\}$ are the original coordinates in Eq. (14) and $\{X\}$ are the multiblade coordinates. A convenient stage to apply the multiblade coordinates is after the process of linearization. In such cases for rotors that have three blades or more, the multiblade coordinate transformation enables one to replace the periodic system by a constant coefficient approximation, which can be treated by conventional eigenvalue calculation methods. Such an approximation is usually reliable[61,62] for advance ratios $\mu < 0.25$, as indicated by research conducted in the early 1980s. Multiblade coordinates are convenient to use with dynamic inflow.

The use of multiblade coordinate transformation is particularly useful for coupled rotor-fuselage problems in hover. In this case use of these coordinates eliminates the periodic coefficients from the equations of motion, for rotors having three blades or more.[83,94,115]

Next, solution of the coupled rotor-fuselage problem in forward flight is briefly discussed. The equations of motion for this problem have again the mathematical form represented by Eq. (14). For level flight conditions with constant advance ratio μ, blade nonlinear equilibrium, in forward flight, is described by Eq. (14). Helicopter trim and the equilibrium solution are extracted simultaneously using a numerical harmonic balance solution.[116,117] The equations are linearized by writing appropriate perturbation equations. After linearization a multiblade coordinate transformation is introduced. The original equation, Eq. (14), has periodic coefficients with a fundamental nondimensional frequency of unity; however, the transformed system has periodic coefficients with a higher fundamental frequency. These higher-frequency periodic terms have a reduced influence on the behavior of the system and can be ignored in some analyses at low advance ratios. Once the transformation is carried out, the system is rewritten in first-order form

$$\{\dot{x}\} = [A(\psi)]\{x\} \tag{18}$$

where $\{x\}$ contains $\{X\}$ and $\{\dot{X}\}$.

The fundamental frequency of the coefficient matrices depends on the number of rotor blades N_b. For an odd-bladed system the fundamental frequency is N_b per revolution, whereas for an even-bladed system the fundamental frequency is $N_b/2$ per revolution. Stability can now be determined using either an eigenvalue analysis (for hover) or Floquet theory for the periodic problem in forward flight. An approximate stability analysis in forward flight is also possible by performing an eigenanalysis on the constant coefficient portion of the system matrices.

When strong nonlinearities, such as dynamic stall, are present in the equations, direct numerical integration is used to obtain blade response. Representative examples of the solution of blade dynamics in the presence of aerodynamic nonlinearities can be found in Refs. 118 and 119. Finally, it should be mentioned that when using dynamic-stall models the procedure described in this section, consisting of linearizing the perturbation equations about a nonlinear time-dependent equilibrium position and extracting stability information using Floquet theory, might not be reliable.

E. Flap-Lag Problem in Hover and Forward Flight

One of the important contributions of the research conducted in the early 1970s was the fundamental understanding of the importance of the lead-lag degrees of freedom in rotary-wing aeroelasticity. It was shown that the flap-lag type of instability is a result of destabilizing inertia and aerodynamic coupling effects associated with the two bending motions present in this problem. It stems from the low aerodynamic damping in the lead-lag motion.

The best treatment of the flap-lag problem in hover was by Ormiston and Hodges,[120] who used a simple centrally hinged, spring-restrained model of a hingeless blade, similar to that shown in Fig. 16, except that the hinge offset was zero and the blade was torsionally rigid. Using a linear analysis, they have shown that this instability is eliminated by elastic coupling, caused by the pitch setting on the blade, which couples blade bending in and out of the plane of rotation. There was very little awareness of the importance of this effect on hingeless blade stability. The treatment of the flap-lag problem, based on a flexible hingeless blade model,

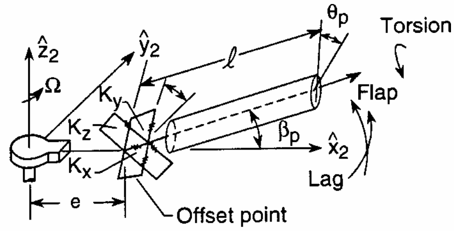

Fig. 16 Offset-hinged spring restrained model of a hingeless blade.

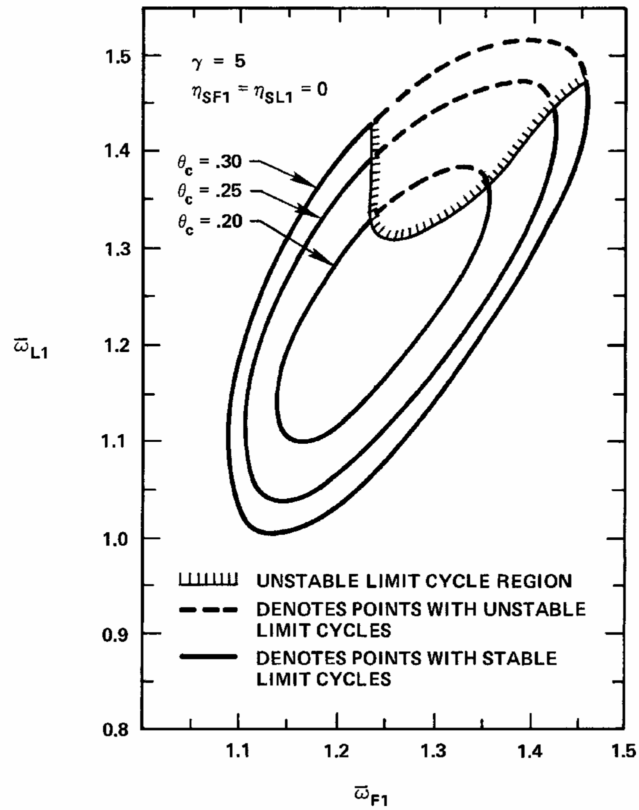

Fig. 17 Typical flap-lag stability boundary in hover without elastic coupling and zero structural damping.

using one elastic mode for each degree of freedom, and a nonlinear analysis conducted in the same time period,[39,121] showed that small amounts of structural damping (1% or less) are also sufficient to eliminate this instability, even when elastic coupling is set equal to zero. Figure 17 depicts a typical stability. Combinations of rotating flap and lag frequencies $\bar{\omega}_{F1}$ and $\bar{\omega}_{L1}$ inside the ellipse-like region represent unstable blade configurations for values of θ_c given on the curves. Here, θ_c is the critical collective pitch setting at which the linearized system becomes unstable. For the fully nonlinear system[121] regions of stable and unstable limit cycles are also shown in Fig. 17. Structural coupling $R_c = 1.0$, or small amounts of damping completely eliminate the unstable, ellipse-like regions for practical values of blade frequencies.

In addition to the theoretical studies on the flap-lag type of instability, Ormiston and Bousman[105] have performed an experimental study that validated the theoretical results. Their findings indicated that under static-stall conditions an unexpected type of blade motion instability for torsionally rigid hingeless rotors was encountered. Elastic coupling was not successful in eliminating this type of instability, as indicated in Fig. 18. Similar conclusions were obtained by Huber.[122]

The flap-lag stability problem, without elastic coupling, exhibits exaggerated sensitivity to small effects that influence the lead-lag damping. For example, although dynamic inflow can affect flap-lag stability when elastic coupling is set equal to zero, with elastic

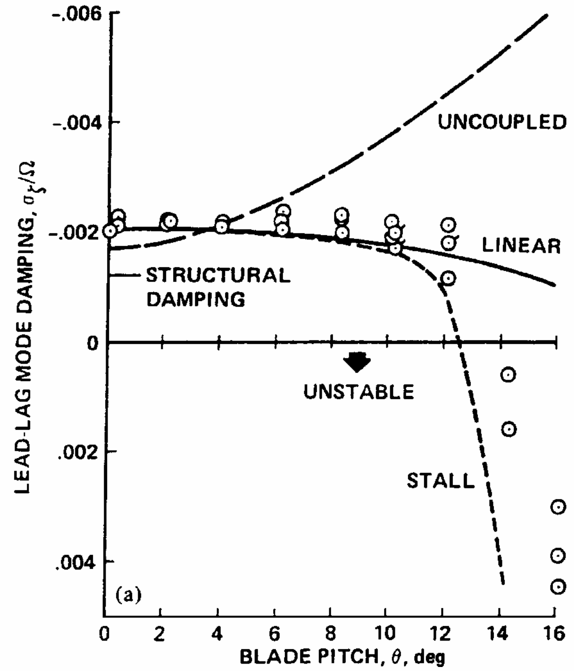

Fig. 18 Stall-induced flap-lag instability; dimensionless lead-lag damping vs collective pitch (300 rpm, R_c = 0.96, $\bar{\omega}_{F1}$ = 1.62, and $\bar{\omega}_{L1}$ = 1.28); *O*, experimental data.

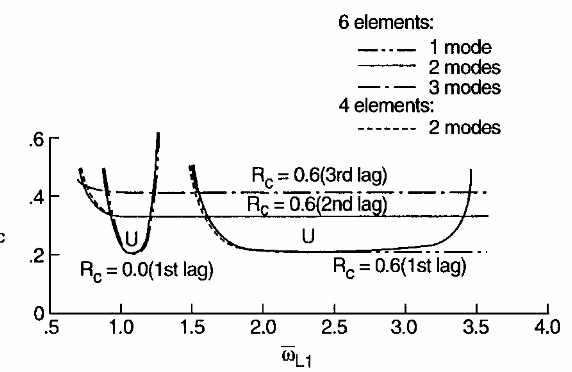

Fig. 19 Convergence of flap-lag stability boundaries in hover, when the number of modes and elements is varied; region above curves is unstable: σ = 0.10, γ = 5.0, and $\bar{\omega}_{F1}$= 1.15.

coupling present the effect of dynamic inflow on this instability in hover is small.[61,117] An interesting result[85] based on a finite element model for a hingeless blade, shown in Fig. 14, is presented in Fig. 19. The blade undergoes only flap and lag motion, and each type of motion is represented by a varying number of elastic global modal degrees of freedom (between 1 and 3, per type of motion): the unstable region of the stability boundaries is denoted by the letter U on the boundary. The figure illustrates the combined effect of the number of modes used in the analysis and the elastic coupling parameter R_c: when $R_c = 1.0$, elastic coupling is present; when $R_c = 0$, elastic coupling is neglected; and values of $0.0 < R_c < 1.0$ represent varying amounts of elastic coupling. From Fig. 19 for $R_c = 0.0$, it is always the fundamental lead-lag mode that yields the lowest stability boundary, and for $R_c = 1.0$ the flap-lag instability is virtually eliminated. The interesting results shown in the figure are the unstable regions associated with the second lag mode, which for intermediate values of the elastic coupling parameter $R_c = 0.60$ becomes unstable at lower values of the critical collective pitch angle than the first lag mode. The implication of this result is that the second elastic lag mode should be retained in the stability analysis of hingeless and bearingless rotor blades.

After understanding the hover problem, several studies on the flap-lag stability problem in forward flight were completed.[106,108,123] These studies were also performed on both the offset-hinged spring restrained model of a hingeless blade[123] as well as the fully elastic blade.[106,108] Among these, Ref. 106 was the most realistic because it contained the effects of trim and reversed flow, which affect blade behavior in forward flight. Later studies have illustrated the sensitivity of flap-lag stability in forward flight[61,62] to dynamic inflow. This sensitivity, which is associated primarily with regressing mode damping, was exaggerated because of the absence of elastic coupling. Subsequently, a theoretical and experimental study[64] has shown that with elastic coupling the effect of dynamic inflow is relatively small.

This description indicates that it took more than a decade to completely understand flap-lag stability in hover and forward flight. In hover it is a fairly weak instability, which can be eliminated by elastic coupling and small amounts of damping. A stronger instability can be triggered on highly loaded rotors operating in stall. In forward flight the problem is sensitive to small terms; and, as will be shown next, in forward flight it is safer to consider the coupled flap-lag-torsional problem.

F. Flap-Lag-Torsional Problem in Hover and Forward Flight

The coupled flap-lag-torsional aeroelastic problem provides a more realistic representation of hingeless blade behavior than the flap-lag problem. However, understanding the flap-lag instability has an important implication for the complete coupled flap-lag-torsional behavior. The first basic studies on coupled flap-lag-torsional aeroelastic behavior of hingeless rotor blades in hover were carried out between 1973 and 1980 (Refs. 54, 104, 122, and 124). This research has shown that soft-in-plane hingeless rotor blades are usually stable. A typical flap-lag-torsional stability boundary taken from Ref. 104 is shown in Fig. 20. The main item of interest in this figure is the bubble-like region of instability present at low values of collective pitch. This instability occurs only in the presence of precone and is a flap-lag type of instability. Sometimes it is called the precone-induced flap-lag instability. It was also obtained in Ref. 54. The unstable region decreases as the fundamental torsional frequency $\bar{\omega}_{\phi 1}$ is increased from 4.5 to 6.0 per revolution. Very small amounts of structural damping in lag ($\eta_{SL1} = 0.0025$, 0.25% of critical in lag) reduce this unstable region for the torsionally soft blade and completely eliminate it for the stiffer blade ($\bar{\omega}_{\phi 1} = 6.0$). Other results not shown here indicate that droop and sweep (see Fig. 3) can have a strong beneficial as well as detrimental effect on the hingeless blade stability. In addition, offsets between cross-sectional elastic axis, aerodynamic center, and center of mass can also influence blade stability. A similar study clarifying the effects

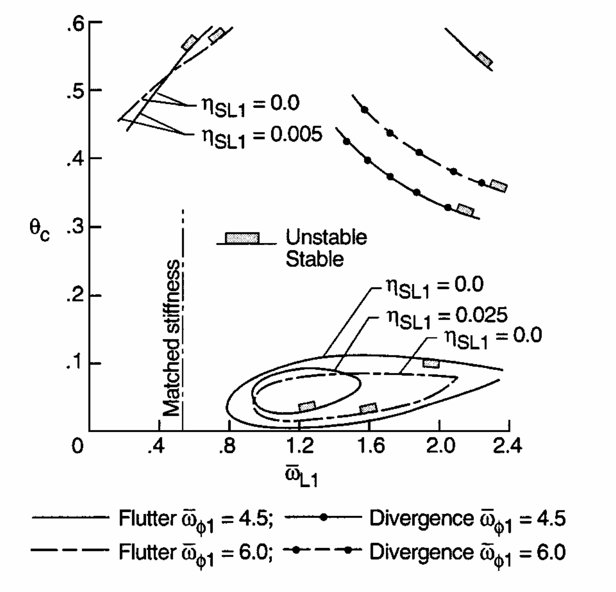

Fig. 20 Coupled flap-lag-torsional stability boundary illustrating combined effect of precone and structural damping for R_c = 1.0, β_p = 3 deg; X_A = 0, γ = 9.0; and $\bar{\omega}_{F1}$ = 1.14.

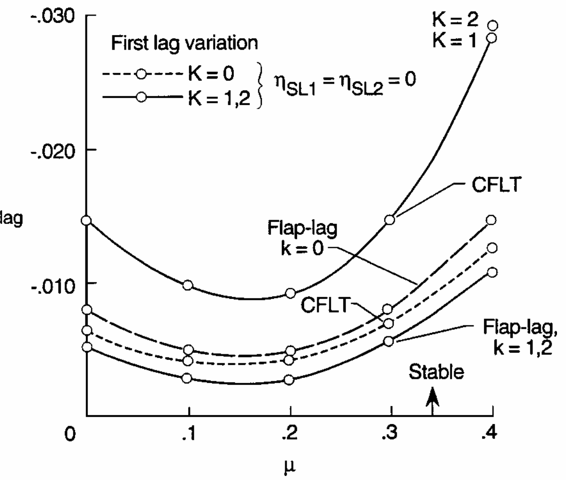

Fig. 21 **Blade stability, lag degree of freedom, soft-in-plane configuration, effect of nonlinear terms, and comparison of flap-lag and flap-lag-torsional analysis (C_W = 0.005, $\bar{\omega}_{L1}$ = 0.732, $\bar{\omega}_{T1}$ = 3.17, γ = 5.5, and σ = 0.07).**

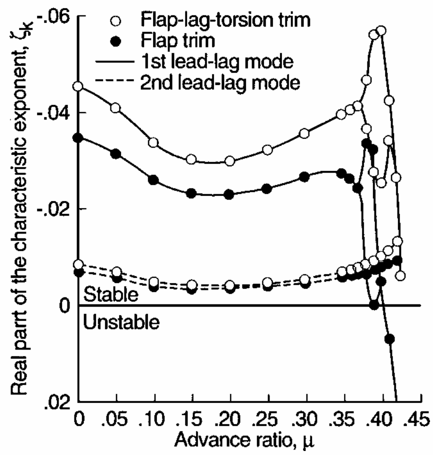

Fig. 22 **Effect of number of degrees of freedom used in trim analysis on lead-lag damping vs μ ($\bar{\omega}_{L1}$ = 1.40, $\bar{\omega}_{T1}$ = 3.0, $\bar{\omega}_{F1}$ = 1.15, σ = 0.10, and R_c = 1.0) in propulsive trim.**

of modeling assumptions on the coupled flap-lag-torsional stability of a stiff-in-plane hingeless blade, including comparisons with experimental data, was conducted in Ref. 125.

Other results, not shown here, also indicate that unsteady aerodynamic effects, wake, and compressibility[53,126] can have a significant effect on the coupled flap-lag-torsional aeroelastic stability of hingeless rotor blades in hover. Another study[127] has shown that by combining three-dimensional tip loss and unsteady inflow effects with a conventional moderate-deflection theory remarkable agreement between theoretical and experimental results was obtained. Most analyses described in this section combine geometrically nonlinear structural models of the blade with *linear* aerodynamic theories. Therefore, an important key to substantial improvements in aeroelastic modeling capability of rotor blades is linked to improving the unsteady aerodynamic models.

Studies aimed at modeling coupled flap-lag-torsional behavior in forward flight started to appear in the early 1980s and continued throughout most of the decade. The first comprehensive study of the coupled flap-lag-torsional dynamics of hingeless rotor blades in forward flight was presented in Ref. 97. This study, based on the equations derived in Ref. 81, clearly demonstrated the role of geometric nonlinearities and trim for this important problem. It was also concluded that usually forward flight is stabilizing for soft-in-plane blade configurations. However, forward flight caused severe degradation in stability of stiff-in-plane configurations.

Figure 21, from Ref. 97, illustrates a number of important effects. The label CFLT on the curves denotes the results from coupled flap-lag-torsional analysis. The label flap-lag denotes results obtained from a flap-lag analysis. The full lines are results from a converged nonlinear analysis, the dashed lines are results for the case when geometrical nonlinearities are neglected. The results shown, depicting the real part of the characteristic exponent for lag as a function of μ, are for propulsive trim. From the figure it is evident that blade stability increases with forward flight for soft-in-plane configurations. The importance of the geometrically nonlinear terms is also evident from the figure. Comparing the stability margin (as represented by ζ_{LAG}) from a flap-lag analysis with that obtained from a flap-lag-torsional analysis it is clear that damping from a flap-lag analysis is 250–300% lower than that obtained from the accurate flap-lag-torsional analysis. Therefore, flap-lag analyses in forward flight can be misleading and should be avoided in trend studies.

Subsequent research on hingeless rotor stability in forward flight,[128] as well as more recent research,[86,129] has confirmed the conclusions presented in Ref. 97. In Refs. 128 and 129 the effect of dynamic inflow was also included and was found to be relatively small. All of the studies mentioned indicated that stiff-in-plane configurations are destabilized by forward flight, whereas the stability of soft-in-plane configurations increases with forward flight.

The results presented in Ref. 97 were based on flap trim only. In Ref. 86, the influence of this approximate trim procedure on blade stability was determined. It is evident from Fig. 22, taken from Ref. 86, that the effect of coupled trim on blade stability is small. The destabilizing effect caused by forward flight on stiff-in-plane rotors is also evident from Fig. 22.

The important trend that emerged from the studies on flap-lag-torsional stability of hingeless blades in forward flight was that forward flight destabilizes stiff-in-plane configurations.

G. Air and Ground Resonance

The ground-resonance problem of articulated rotors has been quite well understood as indicated earlier in this paper. When asymmetry in the rotor support system or in the blades themselves exists, the classical treatment[36] is inadequate. Hammond[130] has considered the ground resonance of an articulated rotor with one lag damper inoperative, using Floquet theory. This was also the first paper to demonstrate the convenient application of Floquet theory to the class of coupled rotor-fuselage problems involving asymmetry. Dissimilarities introduce periodic coefficients in the equations of motion. Blade-to-blade dissimilarities have been considered by McNulty.[131]

The effect of nonlinearities on ground resonance has also been considered. Bellavitta et al.[132] considered the ground resonance of an articulated rotor helicopter where the landing gear had nonlinear characteristics; the solutions were obtained using direct numerical integration. The influence of nonlinear damping on helicopter ground resonance was studied by Tang and Dowell.[133] The analytical model included a three-bladed, articulated rotor, with each blade having only lead-lag motion, combined with a fuselage that could pitch and roll. The formulation contains both a nonlinear damper and a nonlinear landing gear damping. The analytical results were compared with experiments conducted on a model, and good agreement was obtained.

There is also evidence that the aerodynamic loading on the blades can have a significant role on the ground-resonance problem of hingeless and bearingless rotors. For such configurations aeromechanical studies frequently involve both air and ground resonance, which are considered next.

The advent of hingeless and bearingless rotors has generated strong interest in analyses capable of modeling coupled rotor-fuselage problems. Several studies conducted in the late 1970s and 1980s have made important contributions toward the understanding of air-resonance problems.

One of the first comprehensive theoretical studies of the aeromechanical stability of bearingless rotors was conducted by Hodges[134] using the computer program FLAIR, based on the mathematical model described in Refs. 91 and 92. This study[134] and its companion one[135] deal mostly with soft-in-plane configurations using quasi-steady aerodynamics. The analytical results were also compared with experimental data on bearingless main rotors, and good correlation was obtained. FLAIR was also used to study hingeless rotor aeromechanical stability by comparing the theory with experimental data obtained by Bousman.[136] Overall agreement between theoretical predictions and experimental data was good. When FLAIR is used for hingeless rotors, it uses an offset-hinged, spring-restrained model.

Bousman conducted a careful experimental investigation of the effect of elastic couplings on the aeromechanical stability of a hingeless rotor helicopter.[136] Five different configurations were tested to determine to what extent pitch-lag coupling and structural coupling can successfully stabilize the air-resonance mode. This experimental data set has been widely used during the last decade as a basis against which many analytical models have been validated. These experimental results were compared first against theoretical results of Hodges.[91,134] The measured lead-lag regressing mode damping agreed well with theory. Comparison of the theory and experiment for the damping of the body modes showed significant differences that were attributed by Bousman to dynamic inflow.

Johnson[137] compared Bousman's results[136] with analytical predictions of ground resonance, using the model described in Ref. 93. The calculations were performed with and without dynamic inflow. Use of dynamic inflow improved the correlation with experimental data. He obtained the remarkable result that inflow dynamics introduces an additional "inflow mode," which explained previously unresolved questions about the correlation between test and theory.

Venkatesan and Friedmann[83,94] developed a mathematical model for aeromechanical problems associated with multirotor vehicles. A subset of this model consisting of a three-bladed, offset-hinged, spring-restrained model of a hingeless blade with flap-and-flag degrees of freedom for each blade (see Fig. 16) mounted on a gimbal, which could pitch and roll, was used to simulate the experimental data obtained by Bousman.[136] The results obtained using quasi-steady aerodynamics[138] were in good agreement with the experimental data, except that the quasi-steady aerodynamic model was incapable of predicting the dynamic inflow mode found by Johnson.[137] Subsequently both perturbation inflow and dynamic inflow aerodynamics were incorporated in the coupled rotor-fuselage model,[139] and the results obtained with dynamic inflow produced good agreement with the experimental data. Furthermore, the inflow mode obtained by Johnson was also reproduced. Results illustrating this unsteady aerodynamic effect are shown in Figs. 23 and 24. Figure 23 shows the variation of modal frequencies as a function of rotor speed at zero collective pitch setting, using quasi-steady aerodynamics. All frequencies except the one corresponding to 0.7 Hz are predicted well. When perturbation inflow and dynamic inflow are included, the results shown in Fig. 24 indicate that with dynamic inflow all frequencies are predicted well.[140] Furthermore, the inflow mode associated with the augmented states introduced by the dynamic inflow model is also predicted. It is shown in Ref. 139 that the

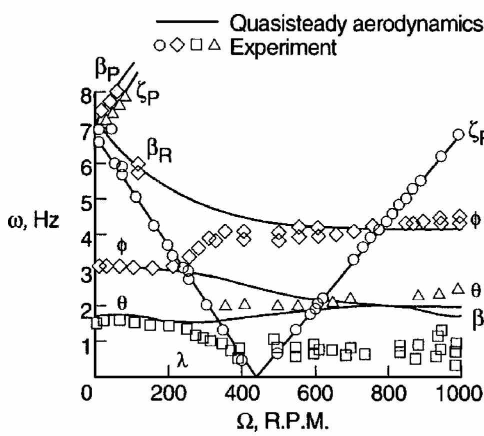

Fig. 23 Variation of modal frequencies with Ω. $\theta_0 = 0$, configuration 4, where ζ_R, regressing lag mode; θ, pitch mode; ϕ, roll mode; and β_R, regressing flap mode.

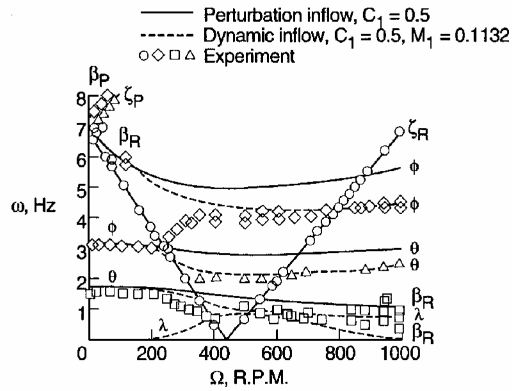

Fig. 24 Variation of modal frequencies with Ω, $\theta_0 = 0$, configuration 4, where λ, inflow mode and all other designations of the curves are identical to Fig. 23.

identification of this mode is relatively complicated. In addition to these results, very good agreement with the regressing mode damping was also obtained.

Another interesting aspect of the coupled rotor-fuselage aeromechanical problem in hover was studied by Loewy and Zotto.[140] They studied the effect of rotor shaft flexibility and associated rotor control coupling on the ground/air resonance of helicopters, which is of interest for certain advanced helicopters that have a relatively flexible shaft. Numerical results were obtained for a four-bladed articulated rotor resembling the OH-58D helicopter. It was found that shaft flexibility/control coupling adds new modes of instability to ground resonance. These models could be easily stabilized by small amounts of structural damping. Air-resonance type of instabilities, however, were found to be more susceptible to shaft flexibility/control coupling, and the instability in this case was stronger.

A comprehensive analytical study of the air-and ground-resonance characteristics of simplified hingeless rotor helicopters was undertaken by Ormiston.[141] The study examined the effect of nonoscillatory body modes on air resonance; the effect of high rotor speeds and high Lock numbers was also considered. The study was restricted to hover.

The studies mentioned were primarily for the case of hover, and they did not clarify the role of the torsional degree of freedom on the air-resonance problem. These items were carefully studied in Refs. 116 and 117. The mathematical model derived for this coupled rotor-fuselage system also has a provision for including an active controller capable of suppressing air resonance. The blade is a simple, offset-hinged, spring-restrained model, with coupled flap-lag-torsional dynamics for each blade attached to a rigid fuselage with five rigid-body degrees of freedom. Unsteady aerodynamics are represented using dynamic inflow of forward flight.[63] In this model there is complete coupling between trim and the aeroelastic analysis. This mathematical model was used to analyze the behavior of a four-bladed hingeless rotor helicopter somewhat similar to the MBB 105 helicopter, with an artificially induced unstable air-resonance mode. The system is described by 37 states. In air-resonance problems the lead-lag regressing mode is the critical degree of freedom. Therefore, the essential features of this instability are described by damping plots for this particular degree of freedom. Figure 25 shows that neglecting the torsional degree of freedom on the nominal configuration increases the instability of the lead-lag regressing mode. The trend of the two curves also tends to diverge at high advance ratios. The addition of torsion amplifies the effect of the periodic terms. At high values of advance ratio, the flap-lag-torsion model shows a much greater difference between the constant and periodic stability analysis than does the flap-lag analysis. The air resonance of hingeless rotors in forward flight was also studied in Ref. 142. Clearly, the neglect of the torsional degree of freedom is not prudent in air-resonance simulations.

It is remarkable that in one decade (1978–1988) considerable progress was made in modeling and understanding air and ground resonance. Reliable models need to include coupled

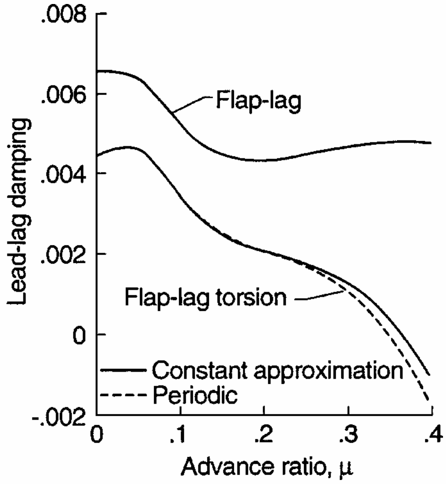

Fig. 25 Effect of torsional flexibility on lead-lag regressing mode damping, air resonance, and soft-in-plane rotor.

flap-lag-torsional blade models with geometric nonlinearities. Fuselage pitch and roll combined with at least two translational degrees of freedom are required. The inclusion of simple unsteady aerodynamics, as represented by dynamic inflow, is also essential. For certain configurations pitch link flexibility and shaft bending might also be important.

H. Composite Blade Modeling

Development of structural dynamic and aeroelastic models for composite blades undergoing moderate or large deflections, along with their application to aeroelasticity of hingeless, bearingless, and tilt-rotor blades, as well as of coupled rotor-fuselage problems, has been a particularly active area of research. Because of its importance, this research topic has also been addressed in several survey papers.[9,143,144] Most modern rotor blades are built from composites because this type of construction guarantees essentially infinite life compared to metal blades used previously that had to be replaced after a few thousand hours of operation.

The first studies on composite blade modeling started to appear in the late 1970s. Mansfield and Sobey[145] initiated the first pioneering study of this difficult subject. They developed the stiffness properties of a fiber-reinforced composite tube subjected to coupled bending, torsion, and extension. Because transverse shear and warping of the cross section were not included in the model, it lacks some of the ingredients necessary for composite rotor-blade aeroelastic analysis.

In the seminal work of Giavotto et al.,[146] prismatic composite beams were modeled making use of the St.-Venant principle, which allowed the "interior" or "central" solutions to be expressed in terms of polynomials in the beam axial coordinate. Using the virtual work principle, a two-dimensional, finite element based cross-sectional analysis was then developed that supplies a fully populated 6×6 matrix [see Eq. (21)] of cross-sectional elastic constants (which determines the shear center location) and stress recovery relations for the cross section in terms of stress resultants. The blade analysis of Ref. 147 uses the stiffnesses from this analysis supplemented by those from Ref. 148 to account for the trapeze effect. These works were part of the development of a comprehensive helicopter analysis in Italy. Despite the generality of this approach, it was largely unknown in the United States until the late 1980s.

The first structural model that was actually incorporated and used in an aeroelastic analysis of a composite rotor blade in hover was developed by Hong and Chopra.[149] In this model the blade was treated as a single-cell, laminated box beam composed of an arbitrary layup of composite plies, and the cross-sectional properties were found analytically. The strain-displacement relations for moderate deflections were taken from Hodges and Dowell,[41] which does not include the effect of transverse shear deformations. Each lamina of the laminate was assumed to have orthotropic material properties. The equations of motion were obtained using Hamilton's principle. A finite element model was used to discretize the equations of motion. Subsequently this analysis was extended to the modeling of composite bearingless rotor blades in hover,[150] and a systematic study was carried out to identify the importance of the stiffness coupling terms on blade stability with fiber orientation and for different configurations. In this model the composite flexbeam of the bearingless rotor blade was represented by an I section consisting of three laminates. In addition to aeroelastic stability studies of composite rotor blades in hover, Panda and Chopra[129] also studied the aeroelastic stability and response of hingeless composite rotor blades in forward flight using the structural model presented in Ref. 149. It was found that ply orientation is effective in reducing both blade response and hub shears.

The accuracy of analytical structural modeling was improved by Rehfield and coworkers,[151−154] culminating in the study of the free vibration of composite beams.[155] This model provided insight into the role of couplings and improved upon the model used in Ref. 149. It was used as the blade stiffness model in Refs. 156 and 157 to treat aeroelastic stability for isolated hingeless, composite rotor blades in the hovering flight condition, using a mixed finite element method based on Ref. 158. Parametric studies are presented to investigate the effects of composite elastic coupling and the thrust condition on the aeroelastic stability, especially that of the lightly damped lead-lag mode. The stability of some of the elastically coupled cases studied was sensitive to the nonclassical couplings. When bending-shear coupling was neglected, for example, this led to significant errors, especially at high thrust levels. Another significant effect stems from changes in the equilibrium solution for elastic twist caused by extension-twist coupling. The necessity of including such effects in the blade model for general-purpose analysis was noted.

A more comprehensive analysis was developed by Kosmatka[159] for the structural dynamic modeling of composite advanced prop-fan blades, which, with some modifications, were also suitable for the general modeling of composite rotor blades. The associated cross-sectional stiffness properties and shear center location were obtained from an accompanying linear two-dimensional finite-element model, which takes into account arbitrary cross-sectional geometry and generally orthotropic materials. The initially twisted blade could undergo moderate deflections. Numerical results for frequencies and mode shapes obtained from this structural dynamic model were in good agreement with modal tests on conventional and advanced propellers.[160]

Bauchau and Hong[47,161] developed a series of large-deflection composite beam models that were intended for rotor-blade structural dynamic and aeroelastic analysis. The final version of this theory is capable of modeling naturally curved and twisted beams undergoing large displacements and rotations and small strain and is a precursor to the beam model in DYMORE (see what follows).

Atilgan and Hodges[162] presented a theory for nonhomogenous, anisotropic beams undergoing large global rotation, small local rotation and small strain, using nonlinear-beam kinematics based on Ref. 48. They used a perturbation analysis to obtain a two-dimensional linear cross-sectional analysis governed by a set of equations identical to those of Ref. 146, uncoupled from the nonlinear, one-dimensional global analysis.[163] This work was used[164] to study the aeroelastic stability of rotor blades using a computer program based on Ref. 146 for determining the blade stiffnesses.

Almost all of the work done by 1990 was restricted to closed cross sections. The early 1990s seems to have marked a turning point in the approach to composite blade modeling used by researchers in RWA. Whereas most of the community continued along the lines of modeling blades in terms of simplified geometries (e.g., box-beams, thin walls, etc.), a subset of the community began instead to focus on modeling of realistic blades (principally Hodges and coworkers) and incorporation of those models into comprehensive analyses (Bauchau and coworkers).

A detailed description of all of the currently available composite blade theories is beyond the scope of this paper. However, some background information is necessary to put into perspective how composite blade theories developed from the early 1990s. In the usual approach to beam theory the in- and out-of-plane deformations of the cross-sectional plane (denoted as warping) are either assumed to be small or neglected. Actually, one can only neglect the in-plane warping (Poisson contraction and antielastic deformation) if the stress field is uniaxial. Although isotropic, prismatic beams have uniaxial stress fields, that is not the case for composite beams in general.

In view of the relative smallness of the warping, the earliest structural models for composite rotor blades determined the cross-sectional warping and elastic constants based on linear analyses.[146,160,165,166] The linear, two-dimensional, cross-sectional analysis is developed based on the assumption that it can be uncoupled from the nonlinear, one-dimensional global analysis for the beam. The sectional analysis is then done once at each of several cross sections of a nonuniform beam; the more rapidly the beam section changes in the spanwise direction, the more sectional analyses are necessary.

Unlike the mere assumption that the cross-sectional analysis is uncoupled or linear, with the use of asymptotic methods one can formulate conditions under which these assumptions actually hold, and a basis for extending the analysis beyond classical cases can be found.[167] Indeed, asymptotic analyses have indeed shown that this uncoupling holds for most cases affecting analysis of rotor blades. Necessary conditions include small strain, linearly elastic materials, and $h \ll \ell$, where h is a typical cross-sectional dimension and ℓ is the wavelength of deformation along the beam axis. Even so, sufficiency is more difficult to establish. For example, the trapeze effect, which satisfies the necessary conditions, can only be obtained from a nonlinear cross-sectional analysis. In any case it is clear that the discussion of composite rotor-blade structural modeling can be divided into two categories: cross-sectional modeling approaches and beam structural models that use one-dimensional beam kinetics and kinematics.

Rotor blades are typically modeled as beams for aeroelasticity and dynamics analyses because of the simplicity of beam theory vs other approaches. Whereas three-dimensional finite element modeling has tremendous capabilities, to model rotor blades in that manner would be extremely expensive, requiring millions of degrees of freedom and an immense amount of setup labor. Not just any beam theory is suitable for composite rotor-blade analysis. A typical structural model in this category should at least include geometric nonlinearities and initial twist. Theories in which only strain is assumed to be small[147,158,168] (sometimes referred to as "geometrically exact" theories) are now the ones most promising for general-purpose analysis.

These methods require cross-sectional elastic constants as input, however, and the determination of these constants is precisely where one encounters the most difficulties in composite-blade modeling. For accurate determination of the cross-sectional elastic constants of composite blades, two distinct characteristics must be present: 1) the resulting theory must be elastically coupled and 2) the cross-sectional deformation must be sufficiently general.

The first requirement, to handle elastic coupling, is exhibited in the strain energy per unit length, a quadratic form involving certain beam generalized strain measures (e.g., the extension of the reference line γ_{11}, the elastic twist κ_1, and the elastic bending curvatures κ_2 and κ_3). For prismatic beams made of isotropic materials, this quadratic form can take a simple form, namely,

$$2U = \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}^T \begin{bmatrix} EA & 0 & 0 & 0 \\ 0 & GJ & 0 & 0 \\ 0 & 0 & EI_2 & 0 \\ 0 & 0 & 0 & EI_3 \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix} \tag{19}$$

When the beam is initially twisted and curved and is made of generally anisotropic materials, the strain energy per unit length instead becomes of the form

$$2U = \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}^T \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix} \tag{20}$$

where the S_{ij} constants depend on initial twist and curvature as well as on the geometry and materials of the cross section. There are two alternative models that are commonly used. When the generalized strains accounting for transverse shear ($2\gamma_{12}$ and $2\gamma_{13}$) are included in the beam model,

$$2U = \begin{Bmatrix} \gamma_{11} \\ 2\gamma_{12} \\ 2\gamma_{13} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}^T \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\ S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ 2\gamma_{12} \\ 2\gamma_{13} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix} \tag{21}$$

When a generalized strain accounting for the Vlasov or restrained warping effect (κ_1') is included in the beam model,

$$2U = \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \\ \kappa_1' \end{Bmatrix}^T \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \\ \kappa_1' \end{Bmatrix} \tag{22}$$

The beam generalized strain measures are, in general, nonlinear functions of the beam displacement and rotational variables.[169]

The second requirement for accurate modeling of composite blades is that the calculation of elastic constants take into account all possible cross-sectional deformations, including transverse shear. Here an important distinction must be made. Because transverse shearing is taken into account in the cross-sectional analysis, this does not imply that one needs explicit transverse shearing generalized strains in the beam strain energy density. Moreover, whether or not a separate warping degree of freedom needs to appear explicitly in the resulting beam theory also depends on the application. The most basic model for closed cross sections, namely, Eq. (20), when its cross-sectional constants are calculated properly, takes transverse shearing into account. Although it has neither transverse shearing nor a separate warping variable, it is sufficiently accurate for analysis of static or low-frequency dynamic behavior.[170] The addition of generalized strain measures for transverse shear deformation [see Eq. (21)] will increase the accuracy of second and higher modal frequencies associated with bending, and retention of a cross-sectional warping variable [see Eq. (22)] will predict more accurately the behavior of thin-walled, open-section beams. It is important to distinguish between interior warping (also called St. Venant warping), which affects the values of the elastic constants of all three of the preceding models, and restrained warping, which is only present in Eq. (22) as an additional generalized strain measure and which changes the boundary conditions of the one-dimensional problem. Indeed, among the most commonly held misconceptions is that one always improves a beam theory by adding more deformation variables in the beam equations, some theories having as many as nine.[171,172] Results obtained from the simplest theory are frequently as good or better, provided the simplest theory has the correct elastic constants.[173] Although there are theories based on ad hoc assumptions that do a good job for certain classes of blade cross sections,[174] the only way to guarantee that a cross-sectional analysis will always predict correct elastic constants is to make certain that it is asymptotically correct in terms of a small parameter, that is, in fact, small. Asymptotically correct means that the approximate solution agrees with an expansion of the exact solution (in this case three-dimensional elasticity) in terms of the small parameter up to a specified power of the small parameter.

Cross-sectional analyses can be classified as either analytical or finite element based. The analytical ones can be further classified as ad hoc or asymptotic. The ad hoc analyses have become quite sophisticated,[174–180] all of which are restricted to the thin-walled case, except Ref. 174. However, asymptotic analyses can yield closed-form results for section constants and stress/strain recovery for beams with thin-walled geometries. As expected, they are the most accurate of the thin-walled analyses, as shown in Refs. 181–184. The ad hoc analyses generally invoke assumptions that do not hold in the general case, such as ignoring the hoop stress or hoop moment or ignoring shell bending measures. The most accurate and powerful of the ad hoc methods appears to be Ref. 174; although it is only applicable to specific cross-sectional geometries, it yields results that compare favorably with those from finite element based analyses. The finite element based analyses can be derived either from the point of view of St. Venant's principle[146,160,165,166,185,186] or from that of asymptotic methods.[167,187] In addition to the variational asymptotic method, which provides a variationally consistent result, there are analyes based on standard asymptotic methods.[188] Cross-sectional analyses are usually linear, but an exception is the trapeze effect, which requires either an initial stress approach[148] or a nonlinear analysis.[189,190]

One computer code for a finite element based cross-sectional analysis that has shown consistently to be quite accurate is the Variational Asymptotic Beam Section Analysis (VABS), originally developed by Cesnik, Yu, Hodges, and their coworkers.[167,170,187,191–196] VABS has promise for meeting industry's requirements for an efficient, reliable analysis tool for composite blades. Validation studies show that it has accuracy and analysis flexibility comparable to more costly, general-purpose three-dimensional finite element analyses such as ABAQUS and can reduce computational effort by two to three orders of magnitude relative to such tools. VABS can perform a classical analysis [i.e., producing a model of the form of Eq. (20)] or a Timoshenko-like analysis [i.e., producing a model of the form of Eq. (21)] for beams with initial twist and curvature. VABS is also capable of capturing the trapeze and Vlasov effects, which are useful for specific beam applications. Finally, VABS can recover the three-dimensional stress and strain fields for finding stress concentrations, interlaminar stresses, etc. VABS is a two-dimensional finite element analysis with a typical element library (triangular elements with 3–6 nodes and quadrilateral elements with 4–9 nodes). It is modular and can be easily integrated into any CAD/CAM software. VABS input is highly compatible with formats used in commercial finite element packages, and so any two-dimensional meshed model of a cross section constructed in PATRAN or ANSYS can be converted into an input for VABS with very little effort.

The last 15 years have exhibited a lot of progress in composite rotor-blade modeling. In summary, currently available composite-blade theories that were developed and have been used in rotary-wing aeroelastic applications can be separated into three groups:

1) Theories in the first group are those in which some ad hoc cross-sectional deformation is assumed, which leads to a set of one-dimensional equations governing behavior of the blade. Although this is the most common approach for blades made of isotropic materials, it can lead to grossly inaccurate results for composite blades. Such assumptions as "plane sections remain plane" or "the cross section is rigid in its own plane" or the uniaxial stress hypothesis can all lead to serious errors. Examples of such errors are presented in Refs. 173 and 197.

2) The second group of theories is based on equations for the blade as a one-dimensional continuum (frequently written in a canonical form), the cross-sectional properties of which are obtained from a separate source. The canonical form of the one-dimensional equations has been known at least since the mid-1980s (Refs. 147, 156, and 168) and typically takes as input a fully populated 6×6 matrix of cross-sectional elastic constants. Methods for finding these constants vary. Unfortunately, this approach lacks a rigorous basis for extension to include effects other than extension, shear, torsion, and bending (such as the Vlasov effect) and nonlinear effects such as the trapeze effect.

3) Theories in the third group are those in which the equations governing cross-sectional deformation and the one-dimensional equations governing behavior of the blade as an equivalent beam are rigorously reduced from the common framework of three-dimensional elasticity theory: This is the newest and most general approach. Examples include Refs. 163, 167, 187, 198–200. It provides the best possible cross-sectional properties, quite accurate strain and stress recovery,[197] and yields the geometrically exact canonical equations of motion for beams.[147,158,168] It has been extended to include such things as the trapeze[190] and Vlasov[184] effects. The trapeze effect accounts for the increase in effective torsional rigidity from axial force, important in rotating beams. The Vlasov effect is important for thin-walled open cross sections, examples of which are typically used in bearingless rotor flexbeams.

These theories have been applied to a large number of rotary-wing aeroelastic problems, as indicated next: 1) aeroelastic behavior of composite hingeless and bearingless rotor blades in hover and forward flight.[156,157,176,180,185,186,201,202] 2) air and ground resonance of helicopters with elastically tailored composite blades[178]; and 3) tilt-rotor aeroelastic performance, stability, and response with elastically coupled composite rotor blades.[203–227]

I. Swept-Tip, Hingeless, and Bearingless Rotors

The preceding sections have provided considerable information on hingeless rotors. Therefore, it is interesting to discuss the aeroelastic behavior of swept tip, or advanced geometry rotors schematically depicted in Fig. 26. "Swept tip" implies both sweep and anhedral. Furthermore, the tip can have a tapered geometry. The structural modeling of swept-tip rotor blades represents an important practical and complicated theoretical problem. An approximate aeroelastic model for swept-tip rotor blades was developed by Tarzanin and Vlaminck[208]; however the approximate representation of the structural, inertia, and aerodynamic coupling effects cause the model to be unreliable. The first consistent model for a hingeless rotor with a swept tips was presented in Refs. 100 and 101. The hingeless blade was modeled using a Galerkin finite element technique, and a special element for structural, inertia, and aerodynamic properties of the swept tip was developed. Both hover and forward flight were considered. It was found that sweep and precone can be used to modify the aeroelastic behavior of the blade. The aeroelastic analysis of swept-tip rotors was also considered in Ref. 209 with transonic aerodynamics and a free wake model. Structural dynamic tests and correlation with a moderate deflection theory were undertaken by Ref. 210. Subsequent correlation by Ref. 211 showed conclusively that geometrically exact analyses correlate much better. Parametric studies for such rotors were carried out.[201] Recently, Maier et al.[212] conducted correlations between experimental data and simulations using the CAMRAD II[213] computer code good agreement was obtained between theory and test for the case of hover. For forward flight the agreement between simulation and test data was considerably worse than for the case of hover.

One of the most important modern rotor systems is the bearingless rotor, schematically depicted in Fig. 4. The analysis of a bearingless main rotor (BMR) is complicated due to the redundant structural configuration in the root region. Mathematical models for such rotors made their first appearance in the late 1970s. One of the earliest analyses of such a rotor configuration was incorporated

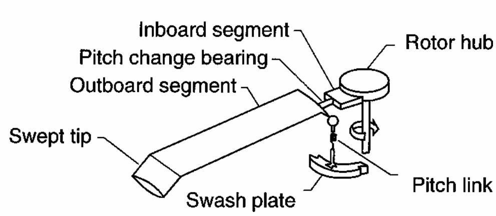

Fig. 26 Typical hingeless blade with advanced geometry tip.

into the FLAIR code.[91,92,134] Subsequently, Sivaneri and Chopra[103] developed a useful finite element model for bearingless rotors. A flexbeam-type bearingless rotor is modeled using regular beam finite elements for the outer portion, a rigid clevis, and multiple beams to represent the flexbeam and the torque tube, as shown in Fig. 27. Special displacement compatibility conditions are enforced at the clevis. This model represents essentially a special redundant root element for the flexbeam.

A section dealing with the properties of hingeless and bearingless rotors would be incomplete without mentioning an insightful study by Weller,[214] which provides a comparison of the aeromechanical stability characteristics, in hover, for two models of conventionally designed soft-in-plane main rotors. One model is a bearingless configuration, simulating the Bell helicopter M680 main rotor. The second model is a hingeless rotor similar to the MBB BO-105 main rotor. The purpose of the study was to compare the test data obtained from the two models, identify their respective aeromechanical stability characteristics, and determine the design features that have a primary effect on the air- and ground-resonance behavior in hover.

In Ref. 214 two Froude-scaled models, one hingeless and one bearingless, were tested. One was an MBB-105 1 : 4 scale rotor, and the other one was a 1 : 4 scale bearingless rotor resembling the Bell bearingless rotor. The rotors were tested on the Advanced Rotorcraft Experimental Dynamics system, which can provide body pitch and roll degrees of freedom, at both low and high thrust conditions. The results obtained indicate that the hingeless rotor concept offers better stability margins at moderate-to-high-thrust conditions because of its aeroelastic characteristics; thus, the hingeless rotor is more stable at 1g thrust and above. For low thrust conditions, however, the bearingless rotor is better because of its larger structural damping caused by the elastomeric lag damper. In these comparisons it is also important to keep in mind that the hingeless rotor had no lag damper, and its damping was caused by its inherent structural damping.

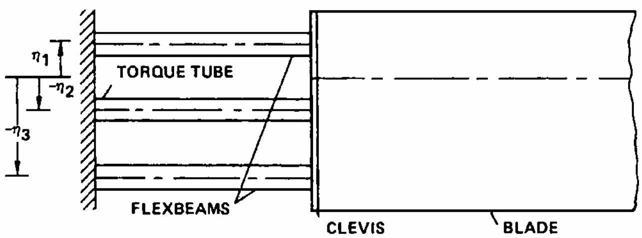

Fig. 27 Idealized finite element model for the root region of a bearingless rotor.

An outstanding study is Ref. 215, which describes in detail the aeroelastic stability wind-tunnel testing of the Comanche BMR and presents correlations with an analytical model. This BMR configuration is depicted in Fig. 28. A series of wind-tunnel tests were performed on a $\frac{1}{6}$ Froude-scaled model of the RAH-66 Comanche BMR at the Boeing vertical/short takeoff and landing wind tunnel. The tests had two objectives: 1) establish the aeromechanical stability characteristics of the coupled rotor-fuselage system and 2) correlate the experimental data with analytical stability predictions so that the methodology can be used with confidence for the full-scale aircraft. An initial test of the rotor with elastomeric dampers, shown in Fig. 29, uncovered a limit-cycle instability. This instability manifested itself for the minimum flight weight configuration. Figure 29, taken from Ref. 215, depicts the frequency and damping of the coupled rotor-body system with elastomeric snubber/dampers. The presence of the body degrees of freedom and their coupling with the blade degrees of freedom modifies significantly the dynamic characteristics compared to the isolated rotor case. A frequency coalescence between the lag regressing and the flap-regressing/body-roll mode now exists. Near this coalescence, the damping is low, and a limit-cycle oscillation occurs at the regressing lag frequency. Closer examination of this nonlinear phenomenon[215] revealed that this problem might also be present when flying with the prototype flight weight. A decision was made to replace the elastomeric snubber/damper by a Fluidlastic® snubber/damper, which is also shown in Fig. 28. The Fluidlastic snubber/damper is similar to the elastomeric dampers, except that it includes a chamber within the flat elements, which is filled with silicone fluid to provide the blade lead-lag damping. As the elastomeric elements that constitute the wall of the chamber flex in shear, the fluid is forced to flow around a rigid diverter protruding into the fluid, thereby generating a damping force.

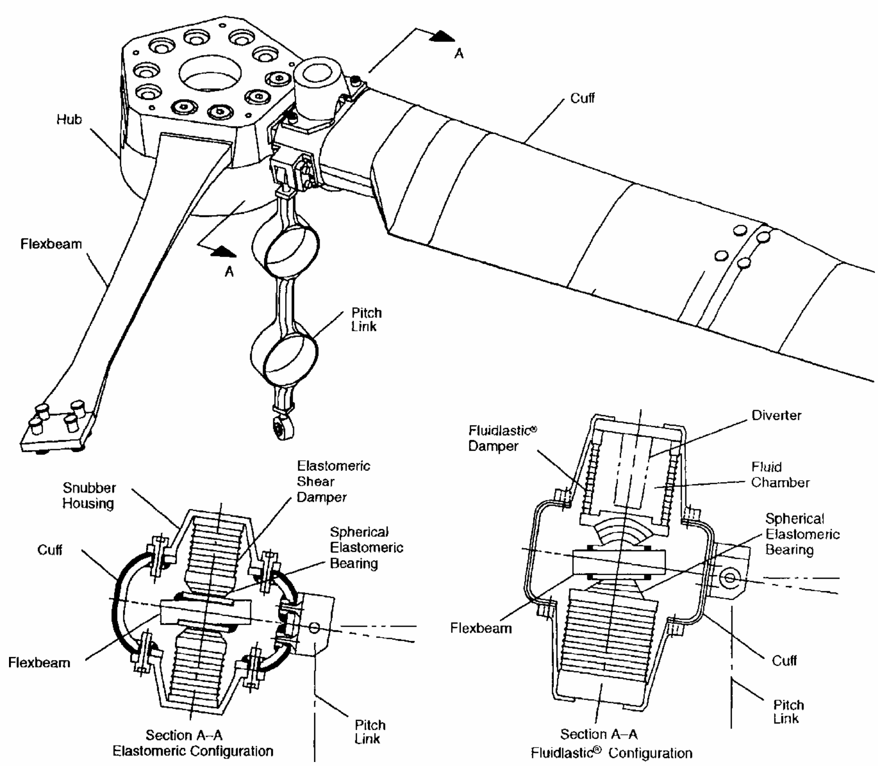

Fig. 28 Description of the Comanche bearingless main rotor, including both elastomeric and Fluidlastic® damper configurations.

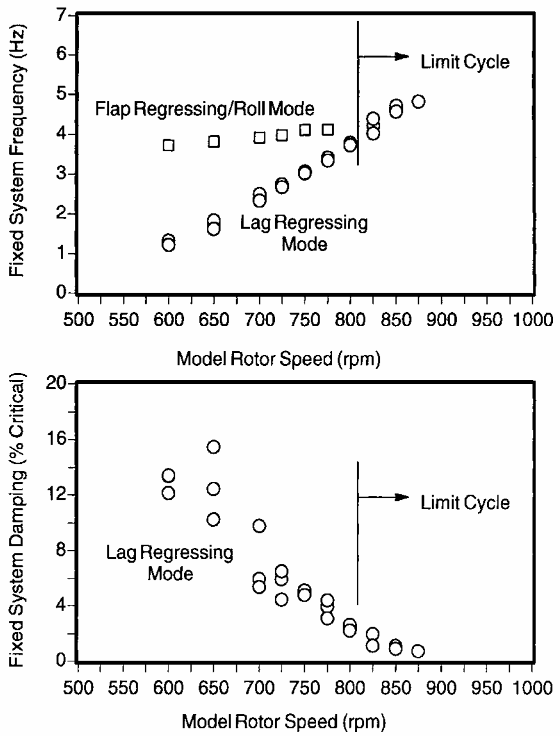

Fig. 29 Hover air resonance of the minimum flight weight configuration with elastomeric dampers at 8-deg collective pitch.

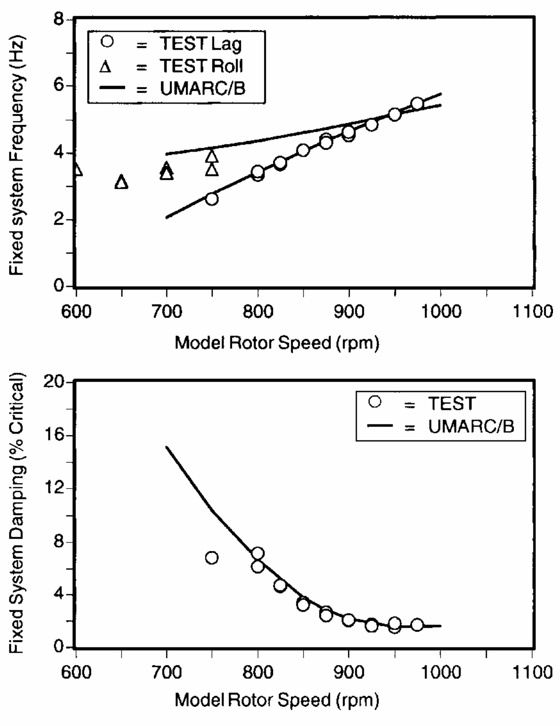

Fig. 30 Hover air resonance at 9-deg collective with Fluidlastic® damper.

Further study revealed that nonlinearities in the stiffness and loss factor of the elastomeric snubber/dampers were the cause of this limit-cycle behavior. As shown in Ref. 215, the stiffness of the Fluidlastic damper is nearly linear, and using it eliminates the limit-cycle instability. Figure 30 shows the hover air response characteristics of the prototype flight weight configuration with the Fluidlastic dampers at 9 deg collective. The test data for both frequency and damping are also compared with analytical results obtained from the UMARC/B code, which is a Boeing modified version of UMARC.[216] The correlations between the results for the code in both hover and forward flight are quite good.

During the last three decades, the helicopter industry in the United States and abroad has invested a very substantial amount of resources in the development of production hingeless and bearingless rotor systems. Hingeless rotored helicopters, such as the MBB BO-105 and the Westland Lynx, have been in production for almost 25 years. However, successful bearingless rotored helicopters have gone into production only during the last decade. Typical examples are the MD-900 Explorer,[217] the Comanche bearingless main rotor (BMR),[215] and the Eurocopter EC135.[218] The MD900 and the Comanche have five-bladed rotors, whereas the EC135 is four-bladed. This is an indication that BMR technology has matured in the last decade, and substantial gains in the understanding of aeroelastic and aeromechanical aspects of these rotors have been made. Therefore, one can view the BMR systems that are currently in production as the crowning achievement of RWA during the last two decades.

J. Comprehensive Analysis Codes

The complexity of the RWA problem has motivated the development of computer codes that have the capability of solving both isolated-blade, as well as coupled rotor-fuselage problems. Once the large effort required was invested, other calculations in the area of performance and flight mechanics were also included in the code. Such codes became known as comprehensive analysis codes. Perhaps the first of these codes, known as REXOR, was developed by Lockheed in the early to mid-1970s (Ref. 219). One of the first successful codes was CAMRAD developed by Johnson,[82,220] which eventually became CAMRAD/JA.[221]

Another important comprehensive analysis code initiated in the early 1980s and completed in the 1990s was the Second Generation Comprehensive Helicopter Analysis System, also known as 2GCHAS,[222–228] which was developed with funding by the U.S. Army Aeroflightdynamics Directorate (formerly known as the Aeromechanics Laboratory), as a second generation replacement for REXOR. Similar codes were developed by various helicopter companies, two of the better known ones are RDYNE[229] and COPTER.[230]

Another very useful code is UMARC,[216] developed at the University of Maryland. The UMAR code developed at the University of Maryland has also enjoyed considerable success, as students who graduated have taken the code with them and started using it in an industrial setting. Subsequently, a more advanced and improved version of CAMRAD/JA was developed: CAMRAD II.[214] The three most advanced, CAMRAD II, 2GCHAS, and DYMORE, have taken advantage of multibody dynamics that facilitate the effective treatment of complex configurations.[231–233]

Among the various comprehensive helicopter analysis codes, CAMRAD II is perhaps the most widely used, both in the United States as well as Europe and Japan. The code has been slightly more successful than its competitors in correlating with experimental data. The wide acceptance of this code is evident from a recent paper that describes the design aspects of a new production bearingless main rotor used on the European EC135. This rotor has excellent damping margins throughout its operation envelope. Modal damping for this rotor in level flight is shown in Fig. 31. The dots are from the flight test, and the solid line is the result of a calculation performed by CAMRAD II. The agreement between theory and test is good. The damping amounts to approximately 2.5% in the rotating system. This rotor is equipped with an elastomeric lag damper and apparently the code reproduces its behavior well.

The 2GCHAS code has also undergone considerable validation during the last five years, and overall the correlations indicate generally satisfactory predictive capability for a fairly wide range of rotorcraft problems. A modified and improved version of the 2GCHAS code has recently become available; it is denoted by the name Rotorcraft Comprehensive Analysis Code (RCAS). In addition to considerable improvements that enhance its computational efficiency and reduce the run times required, the code has the added advantage of being able to run on PC platforms using the Linux operating system.

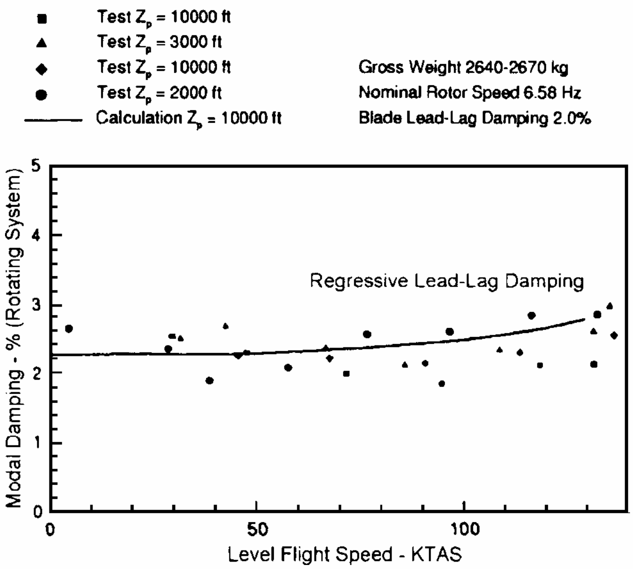

Fig. 31 Regressing lag mode damping in forward flight and comparison with CAMRAD II.

The nonlinear beam element has been significantly improved over the older 2GCHAS code by making use of embedded frames similar to the method used in GRASP.

The multi-flexible-body code DYMORE by Bauchau and coworker has extensive capability in modeling of system hardware.[168,231]

IV. 21st Century—Period of Refinement (2001–present)

It is evident from the various papers published since the turn of the new century that the interest in studies dealing with aeroelastic stability has diminished during the last few years. There is some interest in tilt-rotor aeroelastic stability,[234] aeroelastic scaling,[235] active control for stability augmentation,[234] and aeroelastic analysis of rotors with trailing-edge flaps used for vibration reduction.[236] At the last European Rotorcraft Forum (28th European Rotorcraft Forum, Bristol, United Kingdom, September 2002), there was a total of seven dynamics sessions, where 21 different papers were published and presented. Not a single paper dealt with any significant aspect of the rotary-wing aeroelastic stability problem.

A recent paper[8] clearly indicates that the primary interest has shifted towards active vibration reduction, load correlation, and refinement of existing codes and analyses to provide better agreement with existing experimental databases or new experimental test data generated.

V. Conclusions

This historical perspective illustrates that the progress made in the area of rotary-wing aeroelasticity during the last 60 years has been spectacular. Given the complexity and diversity of the problems that have been considered, as well as those that still persist, it is fair to say that the accomplishments of this relatively short period compare very favorably with what has been accomplished by the fixed-wing community during an entire century. Furthermore, considering the disparity in funding between these two areas of endeavor over such an extended time period further amplifies the accomplishments in RWA.

Acknowledgments

This paper is dedicated to the memory R. H. Miller, who passed away on 28 January 2003 at the age of 87. The authors gratefully acknowledge the funding of the U.S. Army Research Office, NASA, and the National Rotorcraft Technology Center over a long time period. The first author is grateful to his student Dan Patt for his invaluable help in preparing this paper. Without his help this paper would have never made the deadline.

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